



21101945

QP CODE: 21101945

Reg No :

Name :

B.Sc DEGREE (CBCS) EXAMINATION, AUGUST 2021**Third Semester****COMPLEMENTARY COURSE - MM3CMT01 - MATHEMATICS - VECTOR CALCULUS,
ANALYTIC GEOMETRY AND ABSTRACT ALGEBRA**

Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance

2017 Admission Onwards

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Time: 3 Hours

Max. Marks : 80

Part A*Answer any ten questions.**Each question carries 2 marks.*

1. Find the velocity and acceleration of the particle whose position is given by $r(t) = \cos 2t \mathbf{i} + 3 \sin t \mathbf{j}$ at $t = 0$.
2. Find the arc length parametrisation of the curve $r(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k}$ with base point $(1, 0, 0)$.
3. Define directional derivative of a function of three variables.
4. Evaluate $\int_C \sqrt{x^2 + y^2} ds$ along the curve $r(t) = 4 \cos t \mathbf{i} + 4 \sin t \mathbf{j} + 3t \mathbf{k}$ for $-2\pi \leq t \leq 2\pi$.
5. Write the component test for conservative vector fields.
6. Find the divergence of the vector field $\mathbf{F} = 2x\mathbf{i} + 3y\mathbf{j} + z^2\mathbf{k}$ at the point $(1, 0, -1)$.
7. Define an ellipse. Also define foci of the ellipse.
8. Define the asymptotes of a hyperbola. State the equation of the asymptotes of a standard hyperbola.
9. Find the eccentricity of the ellipse $2x^2 + y^2 = 4$.
10. Write an example of a group which is non commutative.



11. Find the number of subgroups of Z_{13} under addition modulo 13.
12. Prove that $f(x) = x + 1$ is a permutation on R .

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Find the derivative of $f(x, y) = x^2 + xy$ at $(1, 2)$ in the direction of the vector $u = i + j$.
14. Find the equation of tangent to curve $x^2 + y^2 = 2$ at the point $(\sqrt{2}, \sqrt{2})$.
15. Use Green's Theorem to find the counterclockwise circulation and outward flux for the field $\mathbf{F} = (x^2 + 4y) i + (y^2 + x) j$ where C is the square bounded by the lines $x = 0, y = 1, x = 1, y = 0$.
16. Find the portion of the plane $z = -x$ inside the cylinder $x^2 + y^2 = 4$.
17. Using Stoke's Theorem, evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = z^2 i + 3xj - y^3 k$ and C is the circle $x^2 + y^2 = 1$ in the xy -plane with counterclockwise orientation looking down the positive z -axis.
18. Express the equation of the curve $x^2 + y^2 + xy = 1$ in polar co-ordinates.
19. Find the focus, equation of the axis and the directrix of the parabola $x = 2y^2$.
20. Let G be a group and H a finite subset of G which is closed under the induced binary operation. Show that H is a subgroup of G .
21. Show that subgroup of a cyclic group is cyclic.

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.

22. (a) Find the unit tangent, principal normal and curvature of the curve $\mathbf{r}(t) = (2t + 3)i + (5 - t^2)j$.
- (b) Find the directions in which $f(x, y, z) = x^3 - xy^2 - z$ increases most rapidly and decreases most rapidly at the point $(1, 1, 0)$.
23. Verify Green's Theorem in the plane for $\mathbf{F} = (3x^2 - 8y^2) i + (4y - 6xy) j$ defined in the region bounded by the lines $x = 0, y = 0, x + y = 1$.
24. (a) Find the center, foci, vertices, directrices and asymptotes of the conic $x^2 - y^2 + 4x - 6y = 6$.



- (b) Find the polar equation of the curve $(x^2 + y^2)^2 - 2ax(x^2 + y^2) - a^2y^2 = 0$.
25. (a) Let H and K be two subgroups of a group G . Then show that $H \cup K$ is a subgroup of G if and only if $H \subset K$ or $K \subset H$.
- (b) Show that if a is a generator of a cyclic group, then a^{-1} is also a generator of that group.

(2×15=30)