QP CODE: 21000690

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, JULY 2021

Fourth Semester

Faculty of Science

Elective - ME800403 - COMBINATORICS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

532D52E3

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight **1** each.

1. Find the number of ways to choose a pair $\{a, b\}$ of distinct numbers from the set $\{1, 2, \ldots, 50\}$ such that $|a - b| \leq 5$
2. State Injection and Bijection Principle
3. How many ways are there to arrange the letters of the word VISITING if no two I's are adjacent
4. Explain Generalized Pigeonhole Principle
5. What is Generalised Ramsay Number
6. State the principle of Inclusion and Exclusion
7. Find the number of integers divisible by 7 in $\{101, 102, \ldots, 400\}$
8. Define $D(n, r, k)$? Write the formula for finding $D(n, r, k)$ $where\ n \geq r \geq k \geq 0 and r \geq 1$
9. What is mean by exponential generating function for a sequence (a_r)?Write the generating function for the sequence $(0!, 1!, 2!, \cdots, r!, \cdots)$.
10. Define r^{th}order linear homogeneous recurrence relation for a sequence (a_n). Give an example of a 3^{rd}order linear homogeneous recurrence relation.

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any **six** questions.

Weight **2** each.

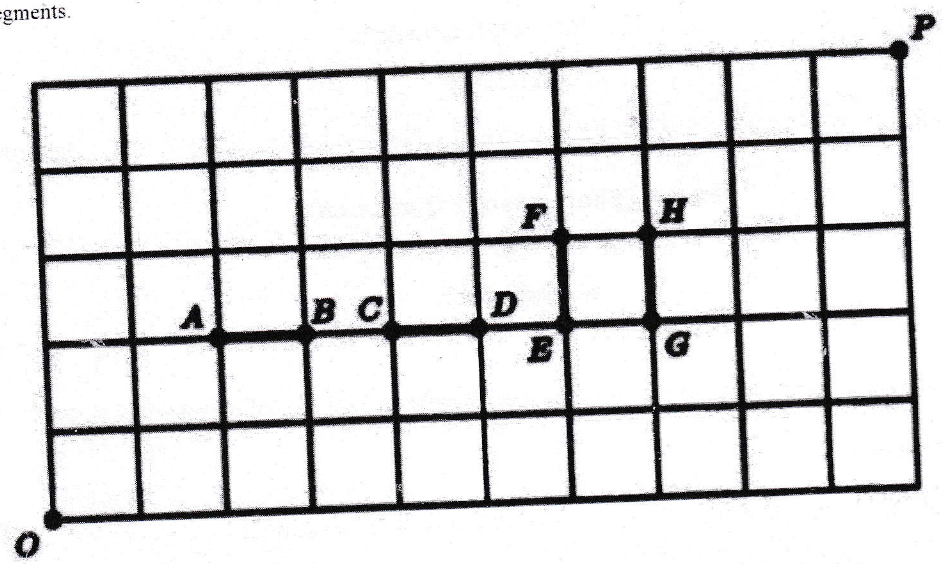
11. A) In how many ways can n+1 different prizes be awarded to n students in such a way that each student has atleast one prize.
    B) In how many ways can 7 boys and 2 girls be lined up in a row such that the girls must seperated by exactly 3 boys

12. Letters of the word "COMMITTEE" is permuted and arranged as in a dictionary.What will be the position of the word COMMITTEE in the dictionary

13. Show that for any set of 17 points chosen within a square whose sides are of length 4 units, there are two points in the set whose distance is at most $\sqrt{2}$ units apart

14. Ten players took part in a round robin chess tournament.According to the rules, a player get +1 if he wins,-1 if he lose and 0 if the game ends in a draw.It was found that more than 70% of the game ends in a draw.Show that there were two players who had the same total score

15. In the 11 by 6 rectangular grid with four specified segments AB, CD, EF and GH. Find the number of shortest routes from O to P in each of the following cases: (i) all the four segments are deleted (ii) Each shortest route must pass through exactly 2 of the four segments.



16. Find the number of surjective maps from $N_n to N_m$

17. Find the coefficient of x^9 *and* x^{14} in the expansion of $(1+x+^2x^3+x^4+x^5)^4$

18. Find the pobability that a roll of 5 dice yields a sum of 17?

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any **two** questions.

Weight **5** each.

19. A)Define Circular permutation and derive a formula for Q^n_n

B)Six boys and 5 irls are seated around a table.Find the number of ways that can be done if

(i) There is no restriction

(ii) No two girls are adjacent

(iii) All girls form a single block

(iv) A particular girl G is adjacent to two particular boys B_1 and B_2

20. A)Six points are there in general position in the space.The 15 line segments joining them in pairs are drawn and painted with some segments with blue and rest of them with red.Prove that there is some triangle has all its sides are of same colour

B) Obtaining a suitable coloring of 5-clique using two colors which is independent of a mono chromatic triangle

21. State and prove generalised Principle of Inclusion and exclusion and hence deduce the formula for $|\bar{A}_1 \cap \bar{A}_2 \ldots \cap \bar{A}_n|$

22. Let a_n denote the number of parallelograms contained in the n^{th} subdivision of an equilateral triangle. Find a recurrence relation for a_n and solve the recurrence relation.

(2×5=10 weightage)