QP CODE: 22000702

22000702

Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2022

Third Semester

Faculty of Science

CORE - ME010304 - FUNCTIONAL ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

705EBAD2

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let X be an n dimensional vector space. Prove that any proper subspace Y of X has dimension less than n.
2. Define convergence and absolute convergence in a normed space.
3. Define a linear operator on a vector space and prove that the range of a linear operator is a vector space.
4. Let X and Y be normed spaces. Show that a linear operator $T : X \to Y$ is bounded if and only if T maps bounded sets in X into bounded sets in Y.
5. Let X and Y be finite dimensional vector spaces over the same field and $T : X \to Y$ be a linear operator. Prove that T determines a unique matrix with respect to a basis for X.
6. Define an inner product space. Give an Example.
7. Let $M \neq \phi$ be a subset of an inner product space X. Show that $M^{\perp}$ is a subspace of X.
8. Define a Total orthonormal set.
9. *Define Hilbert-adjoint operator. Let H_1 and H_2 are Hilbert spaces and $S, T \in B(H_1, H_2)$ then prove that $(S+T)^* = S^* + T^*$*
10. *Prove that the product of two bounded linear operators S and T on a Hilbert space H is self-adjoint if and only if the operators commute ie $ST = TS$.*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Show that $C[a, b]$ is complete.

12. Prove that every finite dimensional subspace Y of a normed space X is complete.

13. Let T be a bounded linear operator. Then prove that

i)if $x_n \to x$, where $x_n, x \in D(T)$ implies $Tx_n \to Tx$

ii) Prove that the Null space of T is closed

14. Prove that there exists a canonical embedding from a vector space X to X^{**}.

15. State and prove Bessel inequality.

16. Let e_k be an orthonormal sequence in a Hilbert space H. Prove that if $\sum_{k=1}^{\infty} \alpha_k e_k$ converges, then $\alpha_k = \langle x, e_k \rangle$, where $x = \sum_{k=1}^{\infty} \alpha_k e_k$.

17. *State Zorn's lemma. Using Zorn's lemma, prove that in every Hilbert space $H \neq \{0\}$, there exists a total orthonormal set.*

18. *Let E be an ordered basis of $\mathbb{R}^n$ and $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear operator. If T is represented by the matrix T_E, then prove that the adjoint operator $T^{\times}$ is represented by the transpose of T_E.*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) Prove that a compact subset M of a metric space X is closed and bounded.

(ii) Prove that a closed and bounded set in a metric space need not be compact.

(iii) Prove that in a finite dimensional normed space X, any subset M of X is compact if and only if M is closed and bounded.

20. i)Show that the dual space of l^1 is l^{∞}

ii)Show that dual space X^l of a normed space X is a Banach space.

21. State and prove Riesz representation theorem.

22. *State and prove Hahn-Banach theorem for extension of linear functionals.*

(2×5=10 weightage)