QP CODE: 22100620

22100620

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, APRIL 2022

Third Semester

Core Course - MM3CRT01 - CALCULUS

Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

805BD574

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Expand e^x using Maclaurin's series.
2. Find the points of inflection of the curve $y = x^3 - 9x^2 + 7x - 6$.
3. Find the radius of curvature at any point on the curve $s = c\tan\psi$
4. Define asymptotes of a curve.
5. If $f(x, y) = x - y$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$
6. State the first derivative test for local extreme values.
7. Write down the Lagrange multipliers equation to find the extreme values of a function $f(x, y, z)$ subject to two constraint $g_1(x, y, z) = 0$ and $g_2(x, y, z) = 0$
8. Evaluate the volume of the solid of cross sectional area $A(x) = 2 + x^2$ from $x = 1$ to $x = 2$.
9. Write down the formula for calculating the volume of solid of revolution about the X-axis and Y-axis.
10. Find the length of the curve $y = x\sqrt{3} + 1$ from $x = 1$ to $x = 2$.
11. Sketch the region of integration and evaluate the integral

$$\int_{\pi}^{2\pi} \int_{0}^{\pi} (\sin x + \cos y)\, dx\, dy$$

12. Define the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the Taylor series generated by f(x) = 1/x at x=2 . Discuss the convergence of the series.

14. Find the equation of the circle of curvature at the point $(0, b)$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

15. Show that the function satisfies Laplace equation $f(x, y) = e^x(x \cos y - y \sin y)$

16. Evaluate $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y} \frac{\partial w}{\partial z}$ in terms of x , y , z if $w = \frac{p - q}{q - r}$, $p = x + y + z$, $q = x - y + z$, $r = x + y - z$,

17. Find the volume of the solid generated when the region under the curve $y = x^2$ over the interval $[0, 2]$ is rotated about the line $y = -1$ using Washer method..

18. Find the area of the surface generated by revolving the curve $x = y^2$; $1 \leq x \leq 9$, about the X-axis.

19. Sketch the region of integration and write an equivalent double integral of $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 5\, x\, dy\, dx$ with the order of integration reversed.

20. Find the volume of the region cut from the cylinder $x^2 + y^2 = 4$ by the planes $z = 0$ and $x + z = 3$.

21. Evaluate the cylindrical coordinate integral $\int_0^{2\pi} \int_0^{\frac{\theta}{2\pi}} \int_0^{3+24r^2} dz\, r\, dr\, d\theta$

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a). Find the evolute of the rectangular hyperbola $xy = 1$

(b). If (X, Y) be the coordinates of centre of curvature of the curve $\sqrt{x} + \sqrt{y} = 1$ at (a, b) , then prove that $X + Y = 3(a + b)$.

23. (a). Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x, y) = x \tan^{-1}(xy)$

(b). Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225$.

24. Compute the volumes of the solid generated by revolving the region bounded by $y = x$ and $y = x^2$ about each coordinate axis using (i) the shell method (ii) the washer method.

25. (a). Evaluate the Jacobian of the transformation from cylindrical coordinate system (r, θ, z) to rectangular system (x, y, z).

(b). Evaluate $\iint_R xy(x^2 + y^2)^{3/2}\, dA$ where R is the region in the first quadrant bounded by the circle $x^2 + y^2 = 1$.

(2×15=30)