22102799

QP CODE: 22102799

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR EXAMINATIONS, AUGUST 2022

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2020 Admission Only

40FC1795

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define an **arc length parameter** for a smooth space curve.
2. Define the curvature of a smooth plane curve. Give a formula for calculating curvature of a smooth plane curve $\mathbf{r}(t)$.
3. Define the **tangent plane** and the **normal line** at a point on a smooth surface in space.
4. Find the accelaration for the position vector $r(t) = (2cost)i + (2sint)j$ at $t = 0$.
5. Find the potential function f for the field $F = 2xi + 3yj + 4zk$.
6. Define flux of a continuous vector field F across an oriented surface S in the positive direction in terms of double integral.
7. Check whether the integer 1729 is an $absolute\ pseudoprime$ or not.
8. Define quadratic congruence with example.
9. Prove $\phi(n) = n - 1$ if and only if n is prime.
10. Define Laplace transform of a function and hence prove that $\mathscr{L}(e^{at}) = \frac{1}{s-a}$.
11. Find $\mathscr{L}^{-1}\left\{\frac{\sqrt{8}}{(s+\sqrt{2})^3}\right\}$.

12. Evaluate $\mathscr{L}(\sin^2 \omega t)$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Give the vector equation and component equation for the plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$. Also find an equation for the plane through $A(0, 0, 1)$, $B(2, 0, 0)$ and $C(0, 3, 0)$.

14. Define derivative for a vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$. If $\mathbf{r}$ is the position vector of a particle moving along a smooth curve in space, then define the particle's velocity vector, direction of motion, speed and acceleration vector.

15. Find the flux of the field $F = (x + y)i - (x^2 + y^2)j$ across the triangle with vertices $(1, 0), (0, 1), (-1, 0)$.

16. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$.

17. Prove that for a scalar function $f(x, y, z)$, curl(grad f) $= 0$.

18. Prove: If $ca \equiv cb \pmod{n}$, then $a \equiv b \pmod{\frac{n}{d}}$, where $d = gcd(c, n)$.

19. Derive the congruence: $a^9 \equiv a \pmod{30}$ for all a.

20. Using convolution theorem, solve $y'' + 5y' + 4y = 2\,e^{-2t},\ y(0) = 0,\ y'(0) = 0$.

21. Solve $y(t) + 2e^t \int_0^t e^{-\tau} y(\tau)\, d\tau = t\, e^t$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.

1. Define the gradient vector of a function in the plane. Find an equation for the tangent to the curve $x^2 - y = 1$ at the point $(\sqrt{2}, 1)$.
2. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. In what direction does f change most rapidly at P_0, and what are the rates of change in these directions?

23. a) State Stoke's Theorem.

b) Find the Circulation of the field $F = (x^2 - y)i + 4zj + x^2k$ around the curve C in which the plane $z = 2$ meets the cone $z = \sqrt{x^2 + y^2}$, counterclockwise as viewed from above.

24.

1. State and prove Fermat's theorem.
2. Prove: If p is a prime, then $a^p \equiv a \pmod{p}$ for any integer a.

25.

1. Let $f(t)$, $f'(t)$ be continuous and satisfy the growth restriction for all $t \geq 0$. Let $f''(t)$ be piecewise continuous on every finite interval on the semi-axis $t \geq 0$. Prove that the Laplace transform of $f''(t)$ satisfies $\mathscr{L}(f'') = s^2\mathscr{L}(f) - sf(0) - f'(0)$.
2. Solve the Initial value problem $y'' + 2y' + 2y = 0,\ y(0) = 1,\ y'(0) = -3$.

(2×15=30)