QP CODE: 22103519

22103519

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, NOVEMBER 2022

Fifth Semester

CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

A4D9D311

Time: 3 Hours

Max. Marks : 80

Part A

Answer any ***ten*** *questions.*

Each question carries ***2*** *marks.*

1. Define absolute value function.
2. If $t > 0$ prove that there exist an $n_t \in N$ such that $0 < \dfrac{1}{n_t} < t$
3. Is any intervals are finite set? Justify.
4. Is the number 1.31311311131111... rational? Give proper reasoning.
5. Show that $lim(\frac{1}{3^n}) = 0.$
6. Prove that (n) is divergent.
7. Give an example of an unbounded sequence that has a convergent subsequence. Explain.
8. Prove that $(1+(-1)^n)$ is not Cauchy.
9. Define properly divergent sequences. Give an example.
10. State Raabe's Test
11. State any two conditions for the convergence of the series $\Sigma x_n y_n$.
12. True or False:" The set $A = \{1, 2\}$ has no cluster points". Give justifications.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Suppose S, T are sets such that $S \subset T$ then prove that

    (a.) If T is finite, then S is finite.

    (b) If S is infinite, then T is infinite

14. State and prove the density theorem on rational numbers? Can the same argument be said about irrationals, verify?

15. Prove that the m-tail of a sequence converges if and only if the sequence converges.

16. Let X = (x_n) is a sequence of real numbers and Y = (y_n) is a sequences of non-zero real numbers that converges to x and y $\neq 0$ respectively. Prove that the sequences X/Y converges to x/y.

17. Let $x_1 = 2$ and $x_{n+1} = 2 + \frac{1}{x_n}$. Prove that $\lim(x_n) = 1 + \sqrt{2}$.

18. If Σa_n is convergent, then prove that any series obtained from it by grouping terms is also convergent to the same value.

19. If Σa_n is a convergent series of real numbers then is it necessary that $\Sigma \frac{\sqrt{a_n}}{n}$ is convergent?

20. Evaluate the one-sided limits of the function $h(x) = \frac{1}{(e^{\frac{1}{x}}+1)}$ at $x = 0$.

21. Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, $c \in \mathscr{R}$ be a cluster point of A. If $f(x) \leq g(x)$ for all $x \in A, x \neq c$,Then prove the following

    (a) If $\lim_{x \to c} f = \infty$, then $\lim_{x \to c} g = \infty$.

    (b) If $\lim_{x \to c} g = -\infty$, then $\lim_{x \to c} f = -\infty$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that the set of all real numbers is a complete ordered field?

23. (a) Prove that for any real number a > 0, there exists a sequence (s_n) of real numbers that converges to $\sqrt{a}$.

    (b) What is Euler number. Prove that Euler number lies between 2 and 3.

24. (a) State and prove the Limit Comparison Test for the convergence of series.

(b) Discuss the convergence of

- $\sum_1^\infty \frac{1}{n^2-n+1}$
- $\sum_1^\infty \frac{1}{\sqrt{n+1}}$

25. (a) If $A \subseteq \mathscr{R}$ and $f : A \to \mathscr{R}$ has a limit at $c \in \mathscr{R}$, then prove that f is bounded on some neighborhood of c.

(b) Show that $\lim\limits_{x\to 0} \frac{1}{x}$ does not exist in $\mathscr{R}$.

(c) Evaluate the limit" $\lim\limits_{x\to 0} \cos x$".

(2×15=30)