QP CODE: 22002313

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Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , NOVEMBER 2022

Second Semester

CORE - ME010202 - ADVANCED TOPOLOGY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

A8C335B9

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let X be a T_2 space and x∈X . Let F be a finite subset of X not containing x. Show that there exist disjoint open sets U and V in X such that x∈U and F⊆V
2. Define normality of a space using the continuous extension of function.
3. Define j'th factor of product space and j' th projection
4. Define the term product topology on X, where $X = \prod_{i \in I} X_i$.
5. Define productive property. Explain with an example
6. State embedding lemma.
7. State the Urysohn's metrisation theorem.
8. Is Countable compactness preserved under continuous functions? Justify.
9. Define a net. Give two examples
10. Define homotopy between two functions.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that in a regular space X , a closed set and a compact set which are disjoint can be separated by means of two disjoint open sets.

12. By Stating necessary results prove that if a topological space X is normal then is has a property that for every two mutually disjoint closed subsets A and B of X, there exist a continuous function f on X to unit interval such that f(x)=0 for all x in A and f(x)=1 for all x in B.

13. A family of boxes is given . Prove that their intersection is again a box.

14. Prove that a topological product of spaces is Tychonoff if and only if each coordinate space is so.

15. Suppose $\{Y_i : i \in I\}$is an indexed family of sets, Z is a set and $\{\theta_i : Z \to Y_i | i \in I\}$is a family of functions such that for any set X and any family $\{f_i : X \to Y_i | i \in I\}$of functions , there exists a unique function $e : X \to Z$ satisfying $\theta_i o e = f_i$ for all $i \in I$. Then prove that there exists a bijection h from Z to $\prod Y_i$ such that for each $i \in I$, $\theta_i = \pi_i o h$. Moreover prove that this bijection is unique.

16. Prove that in a second countable topological space, countable compactness and sequential compactness are equivalent.

17. If limits of all nets in a topological space are unique, show that the space is T_2

18. Let $S : D \to X$be a net and x is a point of X. Then prove that if there exists a subnet whose limit is x in X then x is a cluster point of S

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. *(i)Define Extension of a function.Does every continuous function has a continuous extension.Justify ?*
(ii)Suppose a topological space X has the property that for every closed subsets A of X,every continuous real valued function on A has a continuous extension to X .Show that X is normal

20.
(a) Let(X, τ) be the topological product of an indexed family of topological spaces$\{(X_i, \tau_i) : i \in I\}$ and let Y be any topological space. Prove that a function $f : Y \to X$ is continuous with respect to the product topology on Xif and only if for each $i \in I$, the composition $\pi_i \circ f : Y \to X_i$ is continuous.
(b) Show that projection functions are open ?

21. a) Explain the terms distinguish points and evaluation function.
b)Obtain necessary and sufficient condition for the evaluation function of a family of functions to be one-to-one.
c)Let $f_1, f_2, f_3 : R \to R$ be defined by $f_1(x) = cosx, , f_2(x) = sinx, f_3(x) = x$ for $x \in R$. Describe the evaluation maps of the families $\{f_1, f_2\}, \{f_1, f_2, f_3\}, \{f_1, f_3\}$. Which of these families distinguish points.

22. a) Define a cofinal set and and define the term frequently with respect to net.

b) Prove that if a net S converges to x then x is a cluster point of S

c) Suppose $S : D \to X$ be a net and F is a cofinal subset of S. If $S/F : F \to X$ conveges to a point x in X, then prove that x is a cluster point of S

(2×5=10 weightage)