23104287

QP CODE: 23104287

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, JANUARY 2023

Third Semester

Core Course - MM3CRT01 - CALCULUS

Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

08C7E480

Time: 3 Hours | Max. Marks : 80

core

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Expand a^x by Maclaurin's series.
2. Find the points of inflection of the curve y = x^3 - $3x^2$ - 9x +9.
3. Find the radius of curvature at any point on the curve $s = a\log(\tan\psi + \sec\psi)$
4. Define centre of curvature at any point p of a curve.
5. State the three dimensional Laplace Equation .
6. By implicit differentiation find $\frac{dy}{dx}$ if $x^2 - \sin y + 2y = 0$
7. Explain the method of Lagrange multipliers to find the extreme values of a function $f(x, y, z)$ subject to a constraint $g(x, y, z) = 0$
8. The solid lies between planes perpendicular to the X-axis at $x = 0$ and $x = 4$. The cross-sections perpendicular to X-axis are squares with sides run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$. Find the area of cross section \$A(x)\$.
9. Explain Shell formula for finding volume of solid obtained by revolving a bounded region about a vertical line .

10. Give the formula for finding the length of a smooth curve $y = f(x)$ from $x = a$ to $x = b$.

11. Calculate

$$\int\int_R f(x,y)\,dA \text{ where } f(x,y) = xy\cos y \text{ and } R: -1 \le x \le 1;\ 0 \le y \le \pi$$

12. Find the average value of $f(x,y) = \sin(x+y)$ over the rectangle $0 \le x \le \pi;\ 0 \le y \le \pi/2$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Using Taylor series expand $f(x) = \tan^{-1} x$ in powers of $(x - \frac{\pi}{4})$

14. Prove that the asyptotes of the curve $x^2y^2 = c^2(x^2 + y^2)$ are the sides of a square.

15. If $u = (y-z)(z-x)(x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

16. Find all local extreme values and saddle point, if any, of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20.$

17. Find the volume of the solid generated by revolving the region bounded by the lines and curves $x = y^{\frac{3}{2}};\ x = 0,\ y = 2,$ about the Y-axis.

18. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about the X-axis to generate a solid. Find the volume of the solid.

19. Sketch the region of integration and write an equivalent double integral of $\int_0^2 \int_{x^2}^{2x} (4x+3)\,dy\,dx$ with the order of integration reversed.

20. Evaluate the spherical integral $\int_0^{\pi} \int_0^{\pi} \int_0^{2\sin\phi} \rho^2 \sin\phi\,d\rho\,d\phi\,d\theta.$

21. Solve the system $u = 3x + 2y,\ v = x + 4y$ for x and y in terms of u and v . Then find the value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.

(6×5=30)

Part C

Answer any **two** *questions.*

Each question carries **15** *marks.*

22. a) Find all aymptotes of the cubic curve
$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$
b) Find the envelope of the curve $\left(\frac{x}{a}\right)^3 + \left(\frac{y}{b}\right)^3 = 1$ when $a^2 + b^2 = c^2$.

23. (a). Show that the function $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$ satisfies three dimensional Laplace equation.
(b). Find the absolute maximum and minimum values
$f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate in the first quadrant bounded by the lines $x = 0,\ y = 2,\ y = 2x.$

24. (a). Using the shell method to find the volumes of the solids generated by revolving the regions bounded by the lines and curves
$y = x + 2\ ;\ y = x^2$ about (i) the X-axis (ii) the line $x = 2.$
(b). Find the area of the surface generated by revolving the curve
$y = 2\sqrt{x}\ ;\ 1 \leq x\ \leq 2$ about the X-axis.

25. Find the volume of the region enclosed by the surface
$z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

(2×15=30)