23104205

QP CODE: 23104205

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, JANUARY 2023

Third Semester

COMPLEMENTARY COURSE - MM3CMT01 - MATHEMATICS - VECTOR CALCULUS, ANALYTIC GEOMETRY AND ABSTRACT ALGEBRA

Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance

2017 Admission Onwards

E1147D46

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the unit tangent vector of the curve $r(t) = (t\ sint + cost)\ i + (t\ cost - sint)\ j$.
2. Define gradient vector of a function defined on a plane region.
3. Find the gradient of $f(x, y, z) = x^2 + y^2 + z^2$at $(1, 1, 1)$.
4. Test whether the vector field $\mathbf{F} = yzi + xzj + xyk$ is conservative.
5. Define the curl of a vector field in space.
6. Find the divergence of the vector field $\mathbf{F} = x^3\,i + y^3 j + z^3 k$
7. Define a parabola. Also define its focus and directrix.
8. Find the equation of the ellipse with foci $(\pm 1, 0)$ and $b = \sqrt{5}$.
9. Find the eccentricity of the ellipse $2x^2 + y^2 = 4$.
10. Find the number of subgroups of Z_7 under addition modulo 7.

11. Is the function $f(x) = x^2$ is a permutation on R. Justify?.

12. Give non trivial homomorphism from $< Z, + >$to $< 3Z, + >$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. The position vector of a particle in space at time t is $r(t) = (3t + 1)\, i + \sqrt{3}t\, j + t^2\, k$. Find the particles velocity, acceleration and speed of the particle. Also find the angle between velocity acceleration vectors at $t = 0$.

14. Find the equation of tangent to curve $x^2 + y^2 = 2$ at the point $(\sqrt{2}, \sqrt{2})$.

15. Find the line integral of $f(x, y, z) = x + y + z$ over the straight line segment from $(1, 2, 3)$ to $(0, -1, 1)$.

16. Integrate $g(x, y, z) = x + y + z$ over the portion of the plane $2x + 2y + z = 2$ that lies in the first octant.

17. Integrate $g(x, y, z) = x$ over the parabolic cylinder $y = x^2,\ 0 \leq x \leq 2,\ 0 \leq z \leq 3$.

18. Express the equation of the curve $x^2 + y^2 + 6y = 0$ in polar co-ordinates.

19. Find the vertices, focii, length of the semimajor axis and the length of the semiminor axis of the hyperbola $9y^2 - x^2 = 9$.

20. Let S be the set of all real numbers except -1 and * given by $a * b = a + b + ab$. Show that S is a group under the operation *. Also find a solution of the equation $2 * x * 3 = 7$ in the group $< S, * >$.

21. Find all the elements in the cyclic subgroup of $GL(2, R)$ generated by $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) Show that curvature of a circle of radius a is $\frac{1}{a}$.

(b) Find the directions in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly and decreases most rapidly at the point $(1, 1)$.

23. Verify Stoke's Theorem for $\mathbf{F} = xi + z^2j + y^2k$ where S is the plane $x + y + z = 1$ lying in the first octant.

24. (a) Find the center, foci, vertices, directrices and asymptotes of the conic $x^2 - y^2 + 4x - 6y = 6$.
(b) Find the polar equation of the curve $(x^2 + y^2)^2 - 2ax(x^2 + y^2) - a^2y^2 = 0$.

25. (a) Let G is a group with identity e and of even order, show that there is an element $a \in G$ such that $a \neq e$ and $a = a^{-1}$.
(b) Let H and K be two subgroup of a group G. Show that $H \cap K$ is a subgroup of G

(2×15=30)