QP CODE: 23002638

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, MARCH 2023

Third Semester

Faculty of Science

CORE - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

92C16E87

Time: 3 Hours　　　　　　　　　　　　　　　　Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define orthogonal trajectories on the surface of a given system of curves.
2. *Verify that the equation $yz\,dx + xz\,dy + xy\,dz = 0$ is integrable.*
3. Find the general integral of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$.
4. Explain the different types of integrals of a nonlinear partial differential equations of first order.
5. Prove that the equations $p = P(x, y),\ q = Q(x, y)$ are compatible if $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$.
6. If $z = f(x + iy) + g(x - iy)$ where f and g are arbitrary functions, then show that $r + t = 0$.
7. Find the particular integral of $[D^2 - D']z = e^{x+y}$.
8. The PDE $yu_{xx} + xu_{yy} = 0$ is hyperbolic in which quadrants?
9. Define a family of equipotential surfaces and corresponding potential function.
10. Show that that $\psi = \frac{q}{|r - r'|}$ where q is a constant and r and r' are the position vectors of the points (x, y, z) and (x', y', z'), is a solution of the Laplace's equation $\nabla^2\psi = 0$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Find the integral curves of* $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$

12. Eliminate the arbitrary function f from the given equations.

a) $f(x^2+y^2+z^2,\ z^2-2xy)=0$

b) $z=x+y+f(xy)$

13. Find a complete integral of the equation $p^2x+q^2y=z$.

14. Show that the integral surface of the equation $z(1-q^2)=2(px+qy)$ which passes through the line $x=1,\ y=hz+k$ has an equation $(y-kx)^2=z^2\{(1+h^2)x-1\}$.

15. Solve $[D^3-2D^2D'-DD'^2+2D'^3]z=0$.

16. Solve $r-s+2q-z=x^2y^2$.

17. Prove that if $f(x,y,z)=c$ is a family of equipotential surfaces, then $\frac{\nabla^2 f}{|\nabla f|^2}$ is a function of f alone.

18. Prove that the function $\phi=sinxcoshy+2cosxsinhy+x^2-y^2+4xy$ is Harmonic and find the corresponding analytic function $\phi+i\psi$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Prove the following.

a) A necessary and sufficient condition that the Pfaffian differential equation $X\cdot dr=0$ should be integrable is that $X\cdot curlX=0$.

b) Given one integrating factor of the Pfaffian differential equation $X_1dx_1+X_2dx_2+\cdots+X_ndx_n=0$, we can find an infinity of them.

20. Deduce that the general solution of the linear partial differential equation $Pp+Qq=R$ is of the form $F(u,v)=0$ where F is an arbitrary function and $u(x,y,z)=c_1$ and $v(x,y,z)=c_2$ form a solution of the equations $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$. Hence determine the general solution of the partial differential equation $z(xp-yq)=y^2-x^2$.

21. By Jacobi's method, solve $z^2+zu_z-p^2-q^2=0$.

22. Solve the one- dimensional diffusion equation $\frac{\partial^2 z}{\partial x^2}=\frac{1}{k}\frac{\partial z}{\partial t}$ by separating the variables

(2×5=10 weightage)