QP CODE: 23105587

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2023

Sixth Semester

CORE COURSE - MM6CRT02 - GRAPH THEORY AND METRIC SPACES

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

1552A4CE

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define a Graph.
2. When will you say that two graphs are isomorphic?
3. Draw all non-isomorphic complete bipartite graphs with atmost 4 vertices.
4. What do you mean by an underlying simple graph?
5. Define a bridge. Write the relation between order and size of a connected graph.
6. Define Cut vertex, If Pn is a path of length n , where $n \geq 3$, Then identify the cut vertices of Pn
7. Define Euler graph . Give one example.
8. Define a maximal non Hamiltonian graph. Give an example.
9. Prove that in a metric space X, finite intersection of open sets is open.
10. Define Cantor set.
11. Define limit of a sequence in a metric space.
12. Define complete metric space. Give an example.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Let G be a simple graph with n vertices, where $n \geq 2$. Prove that G has two vertices u and v with d(u) = d(v).

14. Define adjacency matrix of a graph. Draw the graph whose adjacency matrix is $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}$.What can you say about the graph if all the entries of the main diagonal are zero?

15. a) Let G be an acyclic graph with n vertices and k connected components, then prove that G has n – k edges.
b) Justify above result using a graph with 8 vertices.

16. Prove that a graph G is connected if and only if it has a spanning tree.

17. If G is a simple graph with n vertices , where n ≥ 3 , and the degree $d(v) \geq \frac{n}{2}$ for every vertex v of G, Then prove that G is Hamiltonian.

18. Prove that A is open if and only if A = int A.

19. a) Define closure of a set in a metric space X.
b) Prove that A is closed if and only if $\bar{A} = A.$

20. Is limit of a sequence, a limit point of the underlying set? Justify with suitable examples.

21. Let X and Y be metric spaces and a mapping of X into Y. Prove that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a)State and prove First theorem of graph theory.
(b)Prove that in any graph G there is an even number of odd vertices.
(c)Prove that it is impossible to have a group of nine people at a party such that each one knows exactly five of the others in the group.

23. a)State and prove Whitney's theorem for 2- connected graphs.
b) Let u and v be two vertices of the 2- connected graph. Then prove that there is a cycle passing through both u and v.

24. a) Define a metric space. Give an example of a metric on **R**.
b) Let X be the collection of all bounded real valued functions on [0,1]. Prove that d defined d(x,y) = $||f - g||$, where $||f||$ = sup{|f(x): x ∈ [0,1]} is a metric on X.

25. (a) State and prove Cantor's Intersection Theorem.
(b) Let X be a complete metric space and Y a subspace of X. Prove that Y is complete if and ony if Y is closed.

(2×15=30)