QP CODE: 23002639

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, MARCH 2023

Third Semester

Faculty of Science

CORE - ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

41917246

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Write the Fourier series generated by a function f of period 2π. Also give its exponential form as well as the formula for its coefficients
2. Define the inetgral transform of f. Also define the Exponential Fourier transform, Fourier Sine transform and Fourier Cosine transform.
3. Let u, v be two real valued functions defined on a subset S of complex plane, such that u, v are differentiable at an interior point $c \in S$. Then, show that, $f(z) - f(c) = \{\nabla u(c) + i\nabla v(c)\}.(z-c) + o(||z-c||)$.
4. Find the Jacobian matrix for the transformation given by the equation $f(x,y) = (e^{3x+y}, xsiny)$.
5. Show that the mean value theorem for functions from $\mathbf{R}$ to $\mathbf{R}$ does not hold for the function $\mathbf{f} : \mathbf{R} \to \mathbf{R}^2$ defined by the equation, $\mathbf{f}(t) = (cost, sint)$.
6. State and prove the relation connecting the Jacobian determinant of a complex valued function with its derivative.
7. Prove that the function $f : R^2 \to R$ defined by $f(x,y) = (y - x^2)(y - 2x^2)$does not have a local maximum or local minimum at $(0,0)$
8. Assume that the second order partial derivatives $D_{i,j}f$ exist in an n- ball $B(a)$and are continuous at a , where a is a stationary point of f. Let $Q(t) = \frac{1}{2}f''a;t) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} D_{i,j}f(a)t_i t_j$ If $Q(t) < 0$ for all $t \neq 0$, prove that $f$has a relative maximum at a.
9. *Write the necessary and sufficient condition for invertibility of* $G'(a)$, $a \in E \subset R^n$ *where* G *is a primitive mapping.*
10. *Define k forms. Write standard presentation of* $\omega = x_1 dx_2 \wedge dx_1 \wedge dx_3 - x_2 dx_3 \wedge dx_2 \wedge dx_1$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that every real valued and continuous function on a compact interval can be uniformly approximated by a polynomial.
12. Prove the existance of convolution integral of$f$ and g for every x in R, If $f \in L(R), g \in L(R)$ and that either f or g is bounded on R. Moreover if the bounded function f or g is continuous on R. Then h is also continuous on R and $h \in L(R)$.
13. a. Show that if $f(x) = ||x||^2$ and $F(t) = f(\mathbf{c} + t\mathbf{u})$, then $F'(t) = 2\mathbf{c}.\mathbf{u} + 2t||\mathbf{u}||^2$.
    b. Calculate all partial derivatives and directional derivatives of $f(\mathbf{x}) = \mathbf{a}.\mathbf{x}; \mathbf{a} \in R^n$ defined on $\mathbf{R}^n$.

14. a. Show that if $\mathbf{f} : S \to \mathbf{R}^m$ is differentiable at an interior point $\mathbf{c} \in S \subseteq \mathbf{R}^n$ then $\mathbf{f}'(\mathbf{c})(\mathbf{v}) = \sum_{k=1}^{n} v_k D_k \mathbf{f}(\mathbf{c})$, where $\mathbf{v} = (v_1, v_2, \dots v_n)$.

b. Show that if f is real valued then $f'(\mathbf{c})(\mathbf{v}) = \nabla f(\mathbf{c}).\mathbf{v}$.

15. Assume that $f = (f_1, f_2, \dots f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in R^n , and that the Jacobian determinant $J_f(a) \neq 0$ for some point a in S .Prove that there is an n-ball $B(a)$ on which f is one-one.

16. State inverse function theorem and implicit function theorem .

17. *Define* $k -$ *cell* I^k *in* R^k. *Prove that* $\int_{I^k} f$ *is independent of the order in which the k integrations are carried out.*

18. *Let* γ *be a* $1 -$ *surface in* R^3 *with parameter domain* $[0,1]$ *and* $\omega = xdy + ydx$. *Then prove that* $\int_\gamma \omega$ *depends only on the endpoint of the curve* γ

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. If $\mathcal{F}(f)$ denotes the Fourier Transform of f, then prove that $\mathcal{F}(f * g) = \mathcal{F}(f).\mathcal{F}(g)$.

20. Show that the composition of two differentiable functions is again differentiable. Also express total derivative of the composite function in terms of total derivatives of individual functions.

21. State and prove the sufficient condition for the equality of mixed partial derivatives for a function f at the point c

22. *Suppose* F *is a* C' *mapping of an open set* $E \subset R^n$ *into* R^n, $0 \in E$, $F(0) = 0$ *and* $F'(0)$ *is invertible. Then prove that* F *can be represented locally as a composition of primitive mappings and flips.*

(2×5=10 weightage)