23105589

QP CODE: 23105589

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2023

Sixth Semester

CORE COURSE - MM6CRT03 - COMPLEX ANALYSIS

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

5C47794D

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Check whether the function $f(z)=xy^2+i(2x-y)$ is continuous at the origin
2. Verify the analiticity of f(z)=sinz using CR Equations.
3. Given $U(x,y)=\ln(x^2-y^2)$.Verify whether U is harmonic or not
4. Find i^i and its principal value
5. Find all the complex roots of the equation cos z=3
6. Show that If m and n are integers, $\int_0^{2\pi} e^{im\theta}e^{-in\theta}d\theta = \begin{cases} 0; \text{ when } m \neq n \\ 2\pi; \text{ when } m = n \end{cases}$
7. Evaluate $\displaystyle\int_{|z|<2} \frac{ze^z}{(z^2+9)^5}dz$
8. State Liouville's theorem
9. Derive the Maclaurin series expansion for $f(z) = cosz$, using the definition of $cosz = \frac{e^{iz}+e^{-iz}}{2}$
10. Use Laurent series expansion to show that $\displaystyle\int_C e^{\frac{1}{z}}dz = 2\pi i$ where C is any positively oriented simple closed contour around origin.

11. Is $z = 0$, a pole of $f(z) = \frac{1}{z^2(z+1)}$? If so find its order?

12. Define the improper integral over $-\infty < x < \infty$ and its Cauchy Principal Value.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Let f(z)=U+iV where U and V are functions of x and y ,$z_0=x_0+iy_0$ and $w_0=U_0+iV_0$. Then $\lim_{z\to z_0} f(z) = w_0$ iff $\lim_{(xy)\to(x_0y_0)} U(x,y) = U_0$ and $\lim_{(xy)\to(x_0y_0)} V(x,y) = V_0$

14. Verify CR Equations for the function f(z) = $(z^2 - 2)\, e^{-x}$ (cos y - i sin y)

15. Prove that |exp(-2z)|<1 if and only if Re(z)>0

16. Evaluate $\int_C \frac{z+2}{z} dz$, where C is the semicircle $z = 2e^{i\theta}$, $(\pi \le \theta \le 2\pi)$.

17. Let C be the arc of the circle |z|=2 from z=2 to z=2i that lies in the first quadrant. Then without evaluating the integral show that $\left| \int_C \frac{z+4}{z^3-1} dz \right| \le \frac{6\pi}{7}$.

18. State and prove Cauchy's inequality.

19. Assuming a series expansion of e^z,show that $z \cosh z^2 = \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n)!} dz, |\, z \,| < \infty$

20. Using residues, evaluate $\int_C z^2 sin(\frac{1}{z}) dz$ where C is the unit circle about the origin.

21. State Cauchy's Residue Theorem. Using the theorem, evaluate $\int_C \frac{e^{-z}}{z^2} dz$, where C is the circle $|\, z \,| = 3$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that 1) $\sin^{-1} z = -i[\log\ iz + (1-z^2)^{\frac{1}{2}}]$ .Hence deduce $\tan^{-1}z$

2) Evaluate $\tan^{-1}(1+i)$

23.

- State and prove Cauchy's Integral Formula.
- Find the value of $\int_C \frac{1}{(z^2+4)^2} dz$, where C is the circle $|z-i| = 2$ in the positive sense.

24. a) State and prove a necessary and sufficient condition for convergence of sequence $z_n = x_n + iy_n$ of complex numbers.

b) Using this derive a necessary and sufficient condition for convergence of series $\sum_{n=1}^{\infty} z_n$ of complex numbers, where $z_n = x_n + iy_n$.

c) Prove that if a series of complex numbers converges, then the n^{th} term converges to zero, as n tends to infinity.

25. State and prove a necessary and sufficient condition for an isolated singular point z_0 of a function $f(z)$ to be a pole of order m. Derive the formula for residue at z_0 of $f(z)$. Find the residue at $z = i$ of $f(z) = \frac{(Logz)^3}{z^2+1}$.

(2×15=30)