23105591

QP CODE: 23105591

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, MARCH 2023

Sixth Semester

CORE COURSE - MM6CRT04 - LINEAR ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

A2ADAE33

Time: 3 Hours — Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that if the rows x1,x2,.......,xp are linearly independent, then none can be zero
2. Prove that every square matrix is equivalent to its transpose.
3. Define linearly dependent subset of a vector space V. Prove that { (1,1,0), (2,5,3), (0,1,1)} of R^3 is linearly dependent.
4. Check whether { (1,1,2), (1,2,5), (5,3,4) } is a basis of R^3.
5. Define dimension of a vector space V and Find the dimension of R_n [X]
6. If $f : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $f(a,b) = (b,0)$, prove that $Im\,f = Ker\,f$.
7. Define matrix of f relative to fixed ordered bases of vector spaces V and W where $f : V \to W$ is linear.
8. Determine the transition matrix from the ordered basis $\{(1,-1,1),(1,-2,2),(1,-2,1)\}$ of $\mathbb{R}^3$ to the natural ordered basis of $\mathbb{R}^3$.
9. Define a nilpotent linear mapping f on a vector space V of dimension n over a field F. What is meant by index of nilpotency of f.
10. Find the characteristic polinomial of $\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

11. Define the eigen space and geometric multiplicity associated with the eigen value.

12. Define eigen value of a linear map and the eigen vector associated with it.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. a)Prove that every square matrix can be expressed uniquely as the sum of a symmetric matrix and a skew symmetric matrix

b)If $A = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$ Prove that An = $\begin{bmatrix} cosn\theta & sinn\theta \\ -sinn\theta & cosn\theta \end{bmatrix}$

14. a) If A and B are orthogonal nxn matrices prove that AB is orthogonal.

b) Prove that a real 2x2 matrix is orthogonal if and only if it is of one of the forms

$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$ Where $a^2 + b^2 = 1$.

15. Prove that the set Rn of n tuples (x1,x2...xn) of real numbers is a real vector space.

16. Determine which of the following subsets are subspace of a vector space R^4

a) { (x, y, z, t) : x = 1}

b) { (x, y ,z , t) x = y, z = t }

17. Define linear mapping from a vector space to a vector space. Check whether $f : \mathbb{R}^3 \to \mathbb{R}^3$ given by $f(x, y, z) = (z, -y, x)$ is linear.

18. a) Prove that the linear mapping $f : \mathbb{R}^2 \to \mathbb{R}^3$ given by $f(x, y) = (y,\ 0,\ x)$ is injective but not surjective.

b) If $f : V \to W$ is linear, then prove that the following statements are equivalent: (i) f is injective (ii) $Ker\ f = \{0\}$.

19. a) Define rank and nullity of a linear mapping. Find the rank and nullity of $pr_1 : \mathbb{R}^3 \to \mathbb{R}$ defined by $pr_1(x, y, z) = x$.

b) Let V and W be vector spaces each of dimension n over a field F. If $f : V \to W$ is linear, then prove that f is injective if and only if f is bijective.

20. For the matrix A = $\begin{bmatrix} -2 & 5 & 7 \\ 1 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}$ find a matrix P such that P^{-1} A P is diagonal.

21. For the nXn tridiagonal matrix An = $\begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 2 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 2 & 1 & \cdots & 0 & 0 \\ . & . & . & . & \cdots\cdots & . & . \\ 0 & 0 & 0 & 0 & \cdots & 2 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2 \end{bmatrix}$ Prove that det An=n + 1.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) Reduce the following matrix to row echelon form $\begin{bmatrix} 1 & 2 & 0 & 3 & 1 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 0 & 1 & 1 & 3 \\ 1 & 1 & 1 & 2 & 1 \end{bmatrix}$

b) Prove that by using elementary row operation, a non-zero matrix can be transformed to a row-echelon matrix.

c) Prove that every non-zero matrix A can be transformed to a Hermite matrix by using elementary row operations.

23. a)Show that the matrix $\begin{bmatrix} 1 & 3 & 4 & 7 \\ 2 & 3 & 5 & 8 \\ 1 & 4 & 5 & 9 \end{bmatrix}$ has neither a left inverse nor a right inverse.

b)Define an invertible matrix.Prove that if A and B are invertible matrix , then $(AB)^{-1} = B^{-1}A^{-1}$.

c) Prove that the real 2x2 matrix A= $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is invertible if and only if ad - bc ‡0, in which case ,find its inverse.

d) If $A_1, A_2, \ldots, A_P$ are invertible nxn matrices. Prove that the product $A_1, A_2, \ldots, A_P$ is invertible and that $(A_1, A_2, \ldots A_P)^{-1} = A_P^{-1} \ldots A_2^{-1} A_1^{-1}$

24. a) Let V and W be vector spaces over a field F. If $\{v_1, v_2, \ldots, v_n\}$ is a basis of V and $w_1, w_2, \ldots w_n$ are elements of W (not necessarily distinct) then prove that there is a unique linear mapping $f : V \to W$ such that $(i = 1, 2, \ldots, n) \quad f(v_i) = w_i$.

b) Prove that a linear mapping is completely and uniquely determined by its action on a basis.

c) Prove that two linear mappings $f, g : V \to W$ are equal if and only if they agree on any basis of V.

25. a) Define similar matrices and state whether similar matrices have the same rank. Show that if matrices A, B are similar then so are A', B'.

b) Prove that for every $\vartheta \in \mathbb{R}$, the complex matrices $\begin{bmatrix} cos\,\vartheta & -sin\,\vartheta \\ sin\,\vartheta & cos\,\vartheta \end{bmatrix}$, $\begin{bmatrix} e^{i\vartheta} & 0 \\ 0 & e^{-i\vartheta} \end{bmatrix}$ are similar.

c) Prove that the relation of being similar is an equivalence relation on the set of $n \times n$ matrices.

(2×15=30)