23003109

QP CODE: 23003109

Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2023

First Semester

CORE - ME010101 - ABSTRACT ALGEBRA

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

5053C2A9

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Find all abelian groups upto isomorphism of order 360.*
2. *Let* X *be a* G *– set. Define an orbit in* X *under* G.
3. *Define torsion coefficients of a finite abelian group. Find the torsion coefficients of* $\mathbb{Z}_6 \times \mathbb{Z}_{12} \times \mathbb{Z}_{20}$
4. Find the kernel of the homomorphism $\phi : \mathbb{Z}_{18} \to \mathbb{Z}_{12}$, where $\phi(1) = 10$.
5. Prove that every group of prime-power order is solvable.
6. Prove that no group of order 20 is simple
7. *Compute the evaluation homomorphism* $\phi_2(x^2 + 3)$, $F = E = \mathbb{C}$
8. *Find the sum and product of the polynomials* $f(x) = 2x^2 + 3x + 4$ *and* $g(x) = 3x^2 + 2x + 3$ *in* $\mathbb{Z}_6[x]$
9. Define ring of endomorphisms.
10. Is $\mathbb{Q}[x]/(x^2 - 5x + 6)$ a field? Why?

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Let* H *be a normal subgroup of a group* G. *Prove that the cosets of* H *form a group* G/H *under the binary operation* $(aH)(bH) = (ab)H$

12. *Let* X *be a* G *- set. When we say that* G *acts faithfully on* X? *Show that* G *acts faithfully on* X *if and only if no two distinct elements of* G *have the same action on each element of* X.

13. Prove that any two Sylow p-subgroups of a finite group are conjugate.

14. If H and K are finite subgroups of a group G, then prove that $|HK| = \frac{(|H|)\ (|K|)}{|H \cap K|}$.

15. *Let D be a given integral domain and let S be the subset of D x D given by S={(a,b)/a,b* $\in$ *D, b≠0}. Two elements (a,b) ~(c,d)in S if and only if ad=bc. Show that ~ is an equivlence relation.*

16. State and prove the Eisenstein criterion for irreducibility.

17. Show that $\mathbb{Z}_2G$ is a group algebra where G *is the group* $\{e, a\}$.

18. Prove that a field contains no proper non trivial ideals.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) Prove that a direct product of a finite number of abelian groups forms an abelian group.

    (b) Prove that the group $\mathbb{Z}_m \times \mathbb{Z}_n$ *is cyclic and isomorphic to* $\mathbb{Z}_{mn}$ *if and only if gcd of* m *and* n *is 1.*

20. (a) State and prove Cauchy's theorem.

    (b) Let G be a finite group. Prove that G is a p-group if and only if |G| is a power of p.

21. *State and prove Fermat's little theorem. Find the remainder of* 7^{1000} *when divided by 24*

22. (a) Let $\phi : R \to R'$ be a ring homomorphism with kernel H. Then show that the additive cosets of H form a ring R/H. Also prove
that the map $\mu : R/H \to \phi[R]$ defined by $\mu(a+H) = \phi(a)$ is an isomorphism.

(b) Prove that for a subring H of R, the multiplication of additive cosets of H is well defined by the equation $(a+H)(b+H) = ab+H$
if and only if $ah \in H$ and $hb \in H$ for all $a, b \in R$ and $h \in H$.

(2×5=10 weightage)