QP CODE: 23003111

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2023

First Semester

CORE - ME010103 - BASIC TOPOLOGY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

C9D4CE4C

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Show that every open ball is an open set in a metric space
2. Define convergence of sequences in a topological space. Discuss the convergence of sequence in an indiscrete space
3. Define Product topology in finite case
4. Define neighborhood, interior and interior point.
5. Define homeomorphism from a space X to a space Y.
6. Define quotient topology and quotient space.
7. Let A be a Lindeloff subset of a space X. Prove that $(A, \tau|_A))$ is Lindeloff.
8. Prove that compactness is a weakly hereditary property.
9. Define a path in topological space and hence define a path connected space.
10. Give an example of a Hausdorff space which is not regular.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Show that on a finite set containing n elements there are atmost $2^{2^{n-2}}$ distinct topologies and obtain a sharper upper bound for the number of topologies on the same set*

12. Show that second countability is a hereditary property

13. Let the function $f : X \rightarrow Y$ is $\mathfrak{J} - \mathcal{U}$ continuous, where $\mathfrak{J}, \mathcal{U}$ be topologies on X, Y respectively. Then show that for any closed subset A of Y, $f^{-1}(A)$ is closed in X *if and only if* for all $A \subseteq X$, $f(\bar{A}) \subseteq \overline{f(A)}$.

14. Let X have the weak topology determined by a family $\{f_i : X \rightarrow Y_i / i \in I\}$ of functions where each Y_i is a topological space, I be an index set. Then show that for any space Z, a function $g : Z \rightarrow X$ is continuous if and only if for each $i \in I$, the composite $f_i \circ g : Z \rightarrow Y_i$ is continuous.

15. Prove that continuous image of a connected space is connected.

16. Let C is a connected subset of a space X and D is any set such that $C \subset D \subset \overline{C}$. Prove that D is connected.

17. Show that comb space is connected but not locally connected

18. *Suppose y be an accumulation point of a set A in a T_1 space . Show that every neighbourhood of y contains infinitely many points of A*

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. Prove the three equivalent conditions for a base of a topological space

20. Let X be a set, $\theta : P(X) \rightarrow P(X)$ a function such that (i) θ is an expansive operator (ii) ϕ is a fixed point of θ (iii) θ is idempotent (iv) θ commutes with finite unions. Then Show that there exist a unique topology $\mathfrak{J}$ on X such that θ coincides with the closure operator associated with $\mathfrak{J}$. Conversely prove that every closure operator satisfies these properties (i) to (iv).

21. (a) Prove that the property of being a Lindeloff space is preserved under continuous functions .
(b) Prove that every continuous real valued function on a compact space is bounded and attains its extrema .

22. Explain equivalent conditions of regular space

(2×5=10 weightage)