QP CODE: 23003112

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2023

First Semester

CORE - ME010104 - REAL ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

932638F8

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define total variation. Prove that $0 \leq V_f < \infty$ and $V_f = 0$ if and only if f is a constant on $[a, b]$.
2. Let f and g be complex valued functions defined as follows : $f(t) = e^{2\pi it}$ if $t \in [0,1]$ and $g(t) = e^{4\pi it}$ if $t \in [0,1]$. Then prove that the length of g is twice that of f.
3. *Define a partition P of [a, b] and a refinement of P. Also define common refinement.*
4. *If* $f \in \mathscr{R}(\alpha)$ *on [a,b] then prove that* $cf \in \mathscr{R}(\alpha)$ *for every constant c . Also show that* $\int_a^b cf d\alpha = c\int_a^b f d\alpha$.
5. *State the fundamental theorem of calculus.*
6. Let $S_{m,n} = \dfrac{m}{m+n}; m, n = 1, 2, 3, \ldots$. Show that the limit process cannot be interchanged.
7. Define uniform convergence of sequence of functions..
8. Every convergent sequence is a Cauchy sequence. What about the converse?
9. Prove that every member of an equicontinuous family of functions is uniformly continuous.
10. Define the exponential function $E(z)$ and prove the addition formula.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Show using an example that boundedness of f' is not necessary for f to be of bounded variation.

12. Prove using an example that there exists functions with same graph which are not equivalent.

13. *Prove that all continuous functions on [a, b] are Riemann Stieltjes integrable.*

14. *Suppose φ is a strictly increasing continuous function that maps an interval $[A,B]$ onto $[a, b]$. Suppose α is monotonically increasing on $[a, b]$ and $f \in \mathscr{R}(\alpha)$ on $[a,b]$. Define β and g on $[A,B]$ by $\beta(y)=\alpha(\varphi(y))$, $g(y)=f(\varphi(y))$. Then prove that $g \in \mathscr{R}(\beta)$ and $\int_A^B g d\beta = \int_a^b f d\alpha$.*

15. Show that $\mathscr{C}(X)$, the set of all complex valued continuous functions defined on a compact metric space X, is a complete metric space under the metric induced by the supremum norm.

16. Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathscr{R}(\alpha)$ on $[a, b]$, for $n = 1, 2, 3, \ldots$ and suppose $f_n \to f$ uniformly on $[a, b]$. Then prove that $f \in \mathscr{R}(\alpha)$ on $[a, b]$.

17. Prove that there exits a convergent subsequence for a pointwise bounded sequence of complex functions defined on a countable set.

18. State and prove the theorem concerning an inversion in the order of summation of a double sequence.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. (i) State and prove additive property of total variation.
(ii) Let f be continuous on $[a, b]$. Then prove that f is of bounded variation on $[a, b]$ if an only if , f can be expressed as the difference of two increasing continuous functions.

20. *Establish a necessary and sufficient condition for a real bounded function f to be Riemann - Stieltjes integrable w.r.t an increasing function α over $[a,b]$.*

21. Construct a continuos function from $\phi(x) = |x| \ (-1 \leq x \leq 1)$, on the real line which is nowhere differentiable.

22. If f is a continuous complex function on $[a,\ b]$, prove that there exists a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on $[a,\ b]$.

(2×5=10 weightage)