QP CODE: 23003113

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, APRIL 2023

First Semester

CORE - ME010105 - GRAPH THEORY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

7722383B

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Show that the size of a self complementary graph of order n is $\frac{n(n-1)}{4}$
2. Prove that in a simple graph G union of two distinct paths joining two distinct vertices contains a cycle
3. (a) Define (i) edge connectivity of a graph.
   (ii) r-edge connected graph
   (b) Given an example of a graphs G with $\kappa = 1, \lambda = 2, \delta = 3$.
4. Show that each block of a graph G with at least three vertices is a 2-connected subgraph of G
5. Write a short note on any two particular cases of the connector problem.
6. Draw an Eulerian graph which is not Hamiltonian and a Hamiltonian graph which is not Eulerian.
7. Determine the chromatic number of the Petersen graph.
8. For a simple graph G, prove that $\chi(\overline{G}) \geq \alpha(G)$
9. *In any plane embedding of a planar graph, prove that the number of faces remains the same.*
10. *Write three common features of* K_5 *and* $K_{3,3}$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. a) Show that for any simple graph G, Aut(G)=Aut(G^c).
    b) Let G be a simple connected graph with n vertices such that Aut(G) is isomorphic to symmetric group S_n of degree n. Show that G is a complete graph K_n.

12. Find the order and size of $G_1 \vee G_2$

13. If {x,y} is a 2-edge cut of a graph G, show that every cycle of G that contains x must also contain y.

14. Let ($d_1, d_2, \ldots\ldots\ldots\ldots d_n$) be a sequence of positive integers with $\sum_{i=1}^{n} d_i = 2(n-1)$. Then prove that there exists a tree T with vertex set ($v_1, v_2, \ldots\ldots\ldots\ldots v_n$) and $d(v_i)=d_i$, $1 \leq i \leq n$.

15. Describe the construction of closure by an example and also prove that a graph has one and only one closure.

16. Prove that the 3-critical graphs are just the odd cycles C_{2n+1}.

17. *Prove that a graph is planar if and only if each of its block is planar.*

18. *What is the spectrum of C_n?. Explain.*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (a) State and prove Moon's theorem.
    (b) Show that every tournament T is diconnected or can be made into one by the reorientation of just one arc of T.

20.
    a. Show that the number of edges in a tree on n vertices is n-1 and conversely a connected graph on n vertices and n-1 edges is a tree.
    b. State and prove Jordan's theorem.
    c. Define (i) Diametre of a graph (ii) Radius of a graph (iii) Eccentricity of a vertex (iv) Centre of a graph

21. (a) Prove that a graph G is Eulerian iff each edge e of G belongs to an odd number of cycles of G.
    (b) For a nontrivial connected graph G, prove that the following statements are equivalent
    i. G is Eulerian
    ii. the degree if each vertex of G is an even positive integer
    iii. G is an edge disjoint union of cycles.

22. *Prove that every planar graph is 5 – vertex colorable.*

(2×5=10 weightage)