



# Subdivision Graph, Power and Line Graph of a Soft Graph

Rajesh K. Thumbakara<sup>1</sup> , Bobin George<sup>\*2</sup> and Jinta Jose<sup>3</sup>

<sup>1</sup> Department of Mathematics, M.A. College (Autonomous) (Mahatma Gandhi University), Kothamangalam, India

<sup>2</sup> Department of Mathematics, Pavanatma College (Mahatma Gandhi University), Murickassery, India

<sup>3</sup> Department of Science and Humanities, VJCET (APJ Abdul Kalam Technological University), Vazhakulam, India

\*Corresponding author: [bobingeorge@pavanatmacollege.org](mailto:bobingeorge@pavanatmacollege.org)

Received: September 7, 2021

Accepted: November 28, 2021

**Abstract.** Soft set is a classification of elements of the universe with respect to some given set of parameters. It is a new approach for modeling vagueness and uncertainty. The concept of soft graph is used to provide a parameterized point of view for graphs. In this paper we introduce the concepts of subdivision graph, power and line graph of a soft graph and investigate some of their properties.

**Keywords.** Soft set; Soft graph; Subdivision graph; Power; Line graph

**Mathematics Subject Classification (2020).** 05C99

Copyright © 2022 Rajesh K. Thumbakara, Bobin George and Jinta Jose. *This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.*

## 1. Introduction

The idea of soft sets was first given by D. Molodtsov [7] in 1999. A soft set is a classification of elements of the universe with respect to some given set of parameters. It has been shown that soft set is more general in nature and has more capabilities in handling uncertain information. Many authors like Maji *et al.* [6], [5] have further studied the theory of soft sets and used the theory to solve some decision making problems. In 2014, Thumbakara and George [12] introduced the concept of soft graph to provide a parameterized point of view for graphs. In 2015, Akram and Nawas [2] modified the definition of soft graph. They [1] also defined certain types of soft graphs like regular soft graph, soft tree etc. and also explained the concepts of

soft bridge, soft cut vertex, soft cycle etc. Thenge *et al.* [10] contributed more to connected soft graph. Also they [11] studied about the radius, diameter and center of a soft graph and introduced the concept of degree in soft graph. In 2019, Sarala and Manju [8] introduced the concept of soft bi-partite graph and studied some properties. In 2020, Thenge [9] discussed the concepts of adjacency matrix and incidence matrix of a soft graph. Domination over soft graphs is introduced by Venkatraman and Helen [13]. In this paper, we introduce the concepts of power and line graph of a soft graph. Also, we investigate some properties of them.

## 2. Preliminaries

### 2.1 Graphs

We refer [4] for preliminaries from graphs. A graph  $G$  is a pair  $(V, E)$  of two finite sets:  $V$ , the vertex set of  $G$  which is a nonempty set and  $E$ , the edge set which is possibly empty.  $G$  can also be represented by  $(V(G), E(G))$ . The degree of a vertex  $v$  denoted by  $d(v)$  is the number of edges of  $G$  incident with  $v$ . Let  $H$  be a graph with vertex set  $V(H)$  and edge set  $E(H)$  and  $G$  be a graph with vertex set  $V(G)$  and edge set  $E(G)$ . Then we say that  $H$  is a subgraph of  $G$  if  $V(H) \subseteq V(G)$  and  $E(H) \subseteq E(G)$ . A walk in a graph  $G$  is a finite sequence  $W = v_0 e_1 v_1 e_2 v_2 \dots v_{k-1} e_k v_k$  whose terms are alternately vertices and edges such that for  $1 \leq i \leq k$ , the edge  $e_i$  has ends  $v_{i-1}$  and  $v_i$ . We say this walk as a  $v_0$ - $v_k$  walk. Here  $v_0$  is called origin of the walk and  $v_k$  is called the terminus of the walk. A  $u$ - $v$  walk is called closed or open depending on whether  $u = v$  or  $u \neq v$ . Trivial walk is one containing no edges. The number of edges in the walk is called the length of the walk. If the edges in the walk are distinct then the walk is called a trail. If the vertices of the walk are distinct then that walk  $W$  is called a path. A non-trivial closed trail in a graph  $G$  is called a cycle if its origin and internal vertices are distinct. A trail in  $G$  is called an Euler trail if it includes every edge of  $G$ . A tour of  $G$  is a closed walk of  $G$  which includes every edge of  $G$  at least once. An Euler tour of  $G$  is a tour which includes each edge of  $G$  exactly once. A graph is called Euler if it has an Euler tour. A vertex  $u$  is said to be connected to a vertex  $v$  in a graph  $G$  if there is a path in  $G$  from  $u$  to  $v$ . A graph is said to be connected if every two of its vertices are connected. If  $C(u)$  denote the set of all vertices in  $G$  that are connected to  $u$  then the subgraph of  $G$  induced by  $C(u)$  is called the connected component containing  $u$ , or simply the component containing  $u$ . A Hamiltonian path in a graph  $G$  is a path which contains every vertex of  $G$ . A Hamiltonian cycle in a graph  $G$  is a cycle which contains every vertex of  $G$ . A graph is called Hamiltonian if it has a Hamiltonian cycle. First theorem of graph theory says that "For any graph  $G$  with  $e$  edges and  $n$  vertices,  $\sum_{i=1}^n d(v_i) = 2e$ ". A graph  $G$  is said to be bipartite if the vertex set of  $G$  can be partitioned into two nonempty subsets  $X$  and  $Y$  in such a way that each edge of  $G$  has one end in  $X$  and the other end in  $Y$ . For any two vertices  $u$  and  $v$  in a graph  $G$  which are connected by a path, we define distance between  $u$  and  $v$ , denoted by  $d(u, v)$  or  $d_G(u, v)$  to be the length of a shortest  $u$ - $v$  path. Degree sequence of a graph  $G$  is the list of degree of all vertices of the graph in nonincreasing order, that is, from largest degree to smallest degree.

## 2.2 Soft Set

In 1999 D. Molodstov [7] introduced the concept of soft set. Let  $u$  be an initial universe set and let  $e$  be a set of parameters. A pair  $(F, E)$  is called a Soft set (over  $u$ ) if and only  $F$  is a mapping of  $e$  into the set of all subsets of the set  $u$ . That is,  $F : E \rightarrow P(u)$ . In other words, the soft set is a parametrized family of subsets of the set  $u$ . Every set  $F(\varepsilon)$ ,  $\varepsilon \in E$ , from this family may be considered as the set of  $\varepsilon$ -elements of the set  $(F, E)$ , or as the set of  $\varepsilon$ -approximate elements of the soft set.

## 2.3 Soft Graph

Thumbakara and George [12] defined a soft graph as follows. Let  $G = (V, E)$  be a simple graph and  $A$  be any nonempty set. Let  $R$  be an arbitrary relation between elements of  $A$  and elements of  $v$ . That is  $R \subseteq AXV$ . A mapping  $F : A \rightarrow P(v)$  can be defined as  $F(x) = \{y \in V \mid xRy\}$ . The pair  $(F, A)$  is a soft set over  $v$ . Then  $(F, A)$  is said to be a Soft Graph of  $G$  if the subgraph induced by  $F(x)$  in  $G$  is a connected subgraph of  $G$  for all  $x \in A$ .

Akram and Nawas [2] modified the definition of soft graph as follows. Let  $G^* = (V, E)$  be a simple graph and  $A$  be any nonempty set. Let  $R$  be an arbitrary relation between elements of  $A$  and elements of  $V$ . That is  $R \subseteq AXV$ . A mapping  $F : A \rightarrow P(v)$  can be defined as  $F(x) = \{y \in V \mid xRy\}$ . Also, define a mapping  $K : A \rightarrow P(e)$  by  $K(x) = \{uv \in E \mid \{u, v\} \subseteq F(x)\}$ . The pair  $(F, A)$  is a soft set over  $v$  and the pair  $(K, A)$  is a soft set over  $e$ . Then the 4-tuple  $G = (G^*, F, K, A)$  is called a soft graph if it satisfies the following conditions:

- (i)  $G^* = (V, E)$  is a simple graph,
- (ii)  $A$  is a nonempty set of parameters,
- (iii)  $(F, A)$  is a soft set over  $v$ ,
- (iv)  $(K, A)$  is a soft set over  $E$ ,
- (v)  $(F(A), K(A))$  is a subgraph of  $G^*$  for all  $a \in A$

If we represent  $(F(x), K(x))$  by  $H(x)$  then soft graph  $G$  is also given by  $\{H(x) : x \in A\}$ .

## 3. Part of a Soft Graph and Part Degree

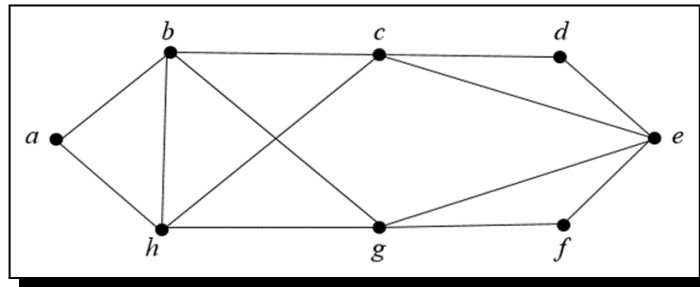
**Definition 3.1** (Part of a Soft Graph). Let  $G^* = (V, E)$  be a graph and  $G = (G^*, F, K, A)$  be a soft graph of  $G^*$  which is also represented by  $\{H(x) : x \in A\}$ . Then  $H(x)$  corresponding to a parameter  $x$  in  $A$  is called a part of the soft graph  $G$ .

**Definition 3.2** (Part Degree). Let  $G = (G^*, F, K, A)$  be a soft graph and  $v$  be any vertex of the part  $H(x)$  of  $G$  for some  $x \in A$ . Then part degree of the vertex  $v$  in  $H(x)$  denoted by  $d(v)[H(x)]$  is the degree of the vertex  $v$  in that part  $H(x)$ .

**Theorem 3.1.** Let  $G^* = (V, E)$  be a graph and  $G = (G^*, F, K, A)$  be a soft graph of  $G^*$  which is represented by  $\{H(x) : x \in A\}$ . Then  $\sum_{x \in A} \sum_{v \in F(x)} d(v)[H(x)] = 2 \sum_{x \in A} |K(x)|$ , where  $|K(x)|$  denotes the number of edges in the part  $H(x)$ .

*Proof.* Let  $H(x) = (F(x), K(x))$  be a part of the soft graph  $G$  for some  $x \in A$ . By applying first theorem of graph theory in the part  $H(x)$  we get  $\sum_{v \in F(x)} d(v)[H(x)] = 2|K(x)|$ . This is true for every part  $H(x)$  of  $G$ . So  $\sum_{x \in A} \sum_{v \in F(x)} d(v)[H(x)] = 2 \sum_{x \in A} |K(x)|$ .  $\square$

**Example 3.1.** Consider a graph  $G^* = (V, E)$  as shown in the following Figure 1.



**Figure 1.** Graph  $G^* = (V, E)$

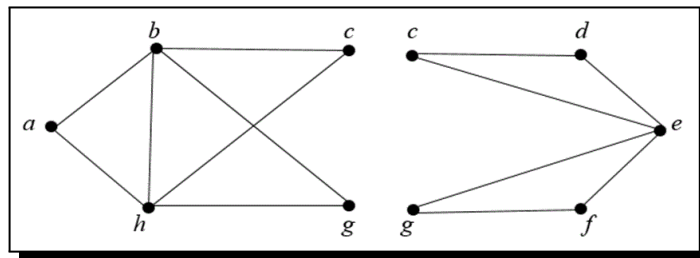
Let  $A = \{b, e\} \subseteq V$  be a parameter set and  $(F, A)$  be a soft set with its approximate function  $F : A \rightarrow P(V)$  defined by  $\{y \in V \mid xRy \Leftrightarrow d(x, y) \leq 1\}$  for all  $x \in A$ .

That is,  $F(b) = \{a, b, c, h, g\}$  and  $F(e) = \{c, d, e, g, f\}$ . Let  $(K, A)$  be a soft set over  $E$  with its approximate function  $K : A \rightarrow P(E)$  defined by  $K(x) = \{uv \in E \mid \{u, v\} \subseteq F(x)\}$  for all  $x \in A$ .

That is,  $K(b) = \{ab, ah, bh, bc, bg, ch, hg\}$  and  $K(e) = \{cd, ce, de, ge, gf, fe\}$ .

Thus  $H(b) = (F(b), K(b))$  and  $H(e) = (F(e), K(e))$  are subgraphs of  $G^*$  as shown in Figure 2.

Hence  $G = \{H(b), H(e)\}$  is a soft graph of  $G^*$ .



**Figure 2.** Soft graph  $G = \{H(b), H(e)\}$

Tabular representation of this soft graph is given in Table 1.

**Table 1.** Tabular representation of the Soft graph  $G = \{H(b), H(e)\}$

$A/V$	$a$	$b$	$c$	$d$	$e$	$f$	$g$	$h$
$b$	1	1	1	0	0	0	1	1
$e$	0	0	1	1	1	1	1	0

$A/E$	$ab$	$ah$	$bc$	$bh$	$bg$	$ch$	$cd$	$ce$	$de$	$ef$	$eg$	$fg$	$gh$
$b$	1	1	1	1	1	1	0	0	0	0	0	0	1
$e$	0	0	0	0	0	0	1	1	1	1	1	1	0

In  $G$  two parts are  $H(b)$  and  $H(e)$ . The part degrees of vertices are calculated as follows:

$d(a)[H(b)] = 2$ ,  $d(b)[H(b)] = 4$ ,  $d(c)[H(b)] = 2$ ,  $d(h)[H(b)] = 4$ ,  $d(g)[H(b)] = 2$ ,  $d(c)[H(e)] = 2$ ,  $d(d)[H(e)] = 2$ ,  $d(e)[H(e)] = 4$ ,  $d(g)[H(e)] = 2$  and  $d(f)[H(e)] = 2$ .

Here

$$\sum_{x \in A} \sum_{v \in F(x)} d(v)[H(x)] = (2 + 4 + 2 + 4 + 2) + (2 + 2 + 4 + 2 + 2) = 14 + 12 = 26.$$

Also

$$2 \sum_{x \in A} |K(x)| = 2(7 + 6) = 26.$$

That is,

$$\sum_{x \in A} \sum_{v \in F(x)} d(v)[H(x)] = 2 \sum_{x \in A} |K(x)| \text{ in } G.$$

#### 4. Subdivision Graph of a Soft Graph

**Definition 4.1** (Subdivision part). Let  $G = (G^*, F, K, A)$  be a soft graph of  $G^*$  represented by  $\{H(x) : x \in A\}$ . Let  $H(x)$  be a part of  $G$  for some  $x \in A$ . Then the subdivision part  $S[H(x)]$  of  $H(x)$  is the graph obtained from  $H(x)$  by replacing each edge  $uv$  of the part  $H(x)$  by a new vertex  $w_{uv}$  and the two new edges  $uw_{uv}$  and  $vw_{uv}$ .

**Definition 4.2** (Subdivision Graph of a Soft Graph). Let  $G = (G^*, F, K, A)$  be a soft graph of  $G^*$  represented by  $\{H(x) : x \in A\}$ . Then subdivision graph of the soft graph  $G$  denoted by  $S(G)$  is given by  $S(G) = \{S[H(x)] : x \in A\}$ , where  $S[H(x)]$  is the subdivision part of  $H(x)$  for  $x \in A$ .

**Example 4.1.** Consider a graph  $G^* = (V, E)$  as shown in the following Figure 3.

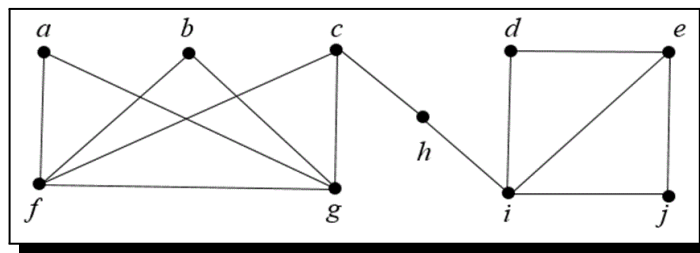


Figure 3. Graph  $G^* = (V, E)$

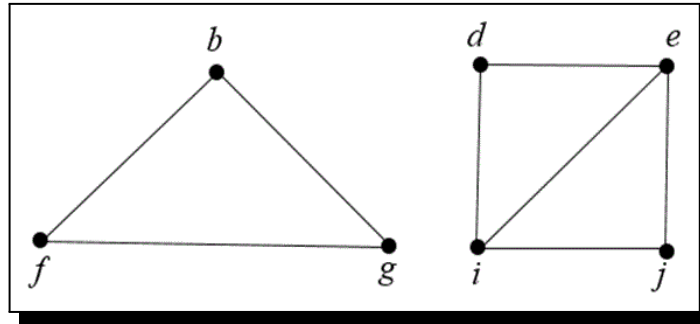
Let  $A = \{b, e\} \subseteq V$  be a parameter set and  $(F, A)$  be a soft set with its approximate function  $F : A \rightarrow P(V)$  defined by  $F(x) = \{y \in V \mid xRy \Leftrightarrow d(x, y) \leq 1\}$  for all  $x \in A$ .

That is,  $F(b) = \{b, f, g\}$  and  $F(e) = \{d, e, i, j\}$ . Let  $(K, A)$  be a soft set over  $E$  with its approximate function  $K : A \rightarrow P(E)$  defined by  $K(x) = \{uv \in E \mid \{u, v\} \subseteq F(x)\}$  for all  $x \in A$ .

That is,  $K(b) = \{bf, bg, fg\}$  and  $K(e) = \{de, di, ei, ej, ij\}$ .

Thus  $H(b) = (F(b), K(b))$  and  $H(e) = (F(e), K(e))$  are subgraphs of  $G^*$  as shown in Figure 4.

Hence  $G = \{H(b), H(e)\}$  is a soft graph of  $G^*$ .



**Figure 4.** Soft graph  $G = \{H(b), H(e)\}$

Tabular representation of this soft graph is given in Table 2.

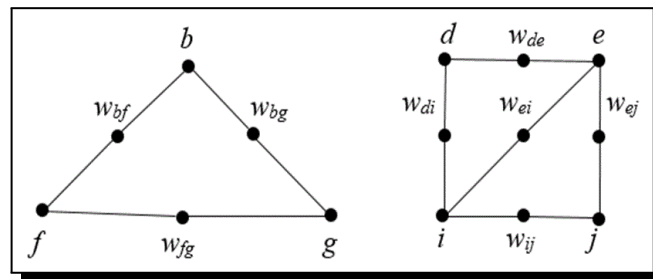
**Table 2.** Tabular representation of the Soft graph  $G = \{H(b), H(e)\}$

A/V	a	b	c	d	e	f	g	h	i	j
b	0	1	0	0	0	1	1	0	0	0
e	0	0	0	1	1	0	0	0	1	1

A/E	af	ag	bf	bg	cf	cg	fg	ch	hi	di	de	ei	ej	ij
b	0	0	1	1	0	0	1	0	0	0	0	0	0	0
e	0	0	0	0	0	0	0	0	0	1	1	1	1	1

Consider this soft graph  $G = \{H(b), H(e)\}$ . Its subdivision graph  $S(G) = \{S[H(b)], S[H(e)]\}$  is given below in Figure 5.



**Figure 5.** Subdivision graph  $S(G) = \{S[H(b)], S[H(e)]\}$

**Theorem 4.1.** Let  $G = (G^*, F, K, A)$  be a soft graph represented by  $\{H(x) : x \in A\}$  and  $S(G) = \{S[H(x)] : x \in A\}$  be the subdivision graph of  $G$ . Then the total number of vertices in  $S(G)$  is  $\sum_{x \in A} (|F(x)| + |K(x)|)$  and the total number of edges in  $S(G)$  is  $2 \sum_{x \in A} |K(x)|$ , where  $|F(x)|$  denotes the number of vertices and  $|K(x)|$  denotes the number of edges in the part  $H(x)$  of  $G$ .

*Proof.* Let  $H(x)$  be a part of the soft graph  $G$  for some  $x \in A$  and  $S[H(x)]$  be the corresponding subdivision part. Then by the definition, the subdivision part  $S[H(x)]$  of  $H(x)$  is the graph obtained from  $H(x)$  by replacing each edge  $uv$  of the part  $H(x)$  by a new vertex  $w_{uv}$  and the two new edges  $uw_{uv}$  and  $vw_{uv}$ . So corresponding to an edge in  $H(x)$ , we have a new vertex in  $S[H(x)]$  and all vertices of  $H(x)$  remain in  $S[H(x)]$ . Also, an edge in  $H(x)$  splits into two edges

in  $S[H(x)]$ . So in the subdivision part  $S[H(x)]$ , total number of vertices is  $(|F(x)| + |K(x)|)$  and the total number of edges is  $2|K(x)|$ . This is true for all subdivision parts  $S[H(x)]$ . Therefore, the total number of vertices in  $S(G)$  is  $\sum_{x \in A} (|F(x)| + |K(x)|)$  and the total number of edges in  $S(G)$  is  $2 \sum_{x \in A} |K(x)|$ .  $\square$

**Example 4.2.** Consider the soft graph  $G = \{H(b), H(e)\}$  given in Figure 4 and its subdivision graph  $S(G) = \{S[H(b)], S[H(e)]\}$  is given in Figure 5. The total number of vertices in  $S(G) = 15 = (3 + 3) + (4 + 5) = \sum_{x \in A} (|F(x)| + |K(x)|)$ . Total number of edges in  $S(G) = 16 = 6 + 10 = 2 \sum_{x \in A} |K(x)|$ .

**Theorem 4.2.** Let  $G = (G^*, F, K, A)$  be a soft graph represented by  $\{H(x) : x \in A\}$  and  $S(G) = \{S[H(x)] : x \in A\}$  be the subdivision graph of  $G$ . Then the subdivision part  $S[H(x)]$  is a bipartite graph for every  $x \in A$ .

*Proof.* Let  $S(x)$  be the subdivision part of  $H(x) = (F(x), K(x))$  for some  $x \in A$ . The subdivision part  $S[H(x)]$  of  $H(x)$  is obtained from  $H(x)$  by replacing each edge  $uv$  of the part  $H(x)$  by a new vertex  $w_{uv}$  and the two new edges  $uw_{uv}$  and  $vw_{uv}$ . So the vertex set of  $S[H(x)]$  denoted by  $V(S[H(x)])$  can be partitioned into two non-empty subsets  $V_1 = F(x)$  and  $V_2 = V(S[H(x)]) - F(x)$  (i.e.,  $V_1 \cup V_2 = V(S[H(x)])$  and  $V_1 \cap V_2 = \emptyset$ ) such that each edge in the subdivision part has one end in  $V_1$  and the other end in  $V_2$ . So  $S[H(x)]$  is a bipartite graph. This is true for every  $x \in A$ .  $\square$

## 5. Power of a Soft Graph

**Definition 5.1** ( $k$ th Power of a Part). Let  $H(x) = (F(x), K(x))$  be any part of a soft graph  $G$  which is connected. Then the  $k$ th power of the part  $H(x)$ , denoted by  $[H(x)]^k$  is defined to be the graph having the same vertex set  $F(x)$  as  $H(x)$  and in which two vertices  $u$  and  $v$  are joined by an edge  $uv$  if and only if in  $H(x)$  we have  $1 \leq d_{H(x)}(u, v) \leq k$ , where  $d_{H(x)}(u, v)$  is the distance between  $u$  and  $v$  in the part  $H(x)$ .

**Definition 5.2** ( $k$ th Power of a Soft Graph). Let  $G = (G^*, F, K, A)$  be a soft graph represented by  $\{H(x) : x \in A\}$  in which all parts  $H(x)$  are connected. Then  $k$ th power of  $G$  denoted by  $G^k$  is defined as  $G^k = \{[H(x)]^k : x \in A\}$ , where  $[H(x)]^k$  is the  $k$ th power of the part  $H(x)$ .  $G^2$  and  $G^3$  are also called square and cube respectively of the soft graph  $G$ .

**Example 5.1.** Consider a graph  $G^* = (V, E)$  as shown in Figure 6.

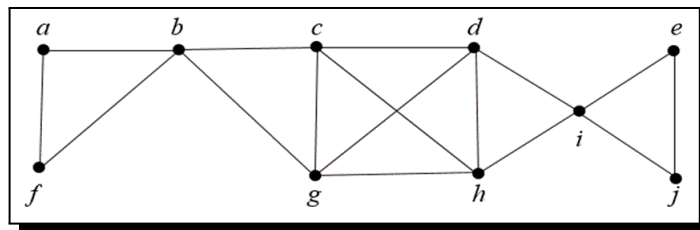


Figure 6. Graph  $G^* = (V, E)$

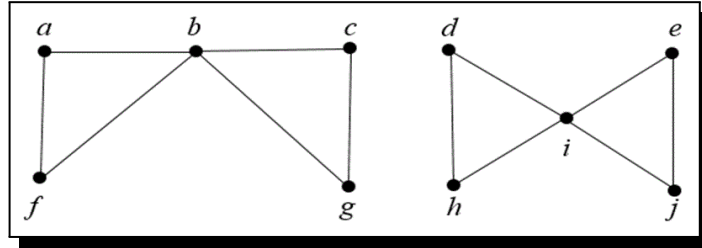
Let  $A = \{a, e\} \subseteq V$  be a parameter set and  $(F, A)$  be a soft set with its approximate function  $F : A \rightarrow P(V)$  defined by for all  $x \in A$ .

That is,  $F(a) = \{a, b, c, f, g\}$  and  $F(e) = \{d, e, h, i, j\}$ . Let  $(K, A)$  be a soft set over  $E$  with its approximate function  $K : A \rightarrow P(E)$  defined by  $K(x) = \{uv \in E \mid \{u, v\} \subseteq F(x)\}$  for all  $x \in A$ .

That is,  $K(a) = \{ab, bc, af, bf, bg, cg\}$  and  $K(e) = \{dh, di, hi, ei, ej, ij\}$ .

Thus  $H(a) = (F(a), K(a))$  and  $H(e) = (F(e), K(e))$  are subgraphs of  $G^*$  as shown in Figure 7.

Hence  $G = \{H(a), H(e)\}$  is a soft graph of  $G^*$ .



**Figure 7.** Soft graph  $G = \{H(a), H(e)\}$

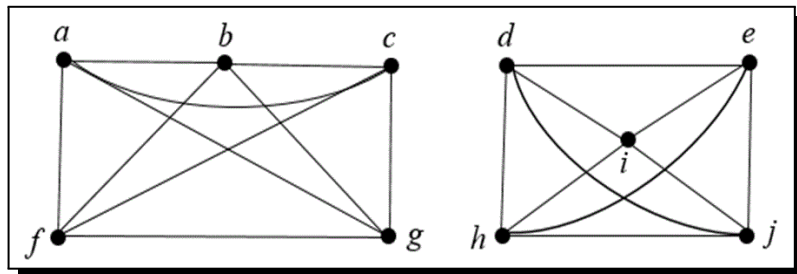
Tabular representation of this soft graph is given in Table 3.

**Table 3.** Tabular representation of the Soft graph  $G = \{H(a), H(e)\}$

$A/V$	$a$	$b$	$c$	$d$	$e$	$f$	$g$	$h$	$i$	$j$
$a$	1	1	1	0	0	1	1	0	0	0
$e$	0	0	0	1	1	0	0	1	1	1

$A/E$	$ab$	$af$	$bc$	$bf$	$bg$	$cg$	$cd$	$ch$	$dg$	$dh$	$di$	$ei$	$ej$	$gh$	$hi$	$ij$
$a$	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
$e$	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1

The square of this soft graph  $G = \{H(a), H(e)\}$  is  $G^2 = \{[H(a)]^2, [H(e)]^2\}$  and is given in Figure 8.



**Figure 8.**  $G^2 = \{[H(a)]^2, [H(e)]^2\}$

**Theorem 5.1.** Let  $G = (G^*, F, K, A)$  be a soft graph represented by  $\{H(x) : x \in A\}$ . If  $H(x)$  is a Hamiltonian part for some  $x \in A$  then  $[H(x)]^k$  will be Hamiltonian.

*Proof.* Consider a Hamiltonian part  $H(x)$  of  $G$ . Then  $[H(x)]^k$  will be Hamiltonian since the supergraph of a Hamiltonian graph is Hamiltonian.  $\square$

## 6. Line Graph of a Soft Graph

**Definition 6.1** (Line Graph of a Part). Let  $G = (G^*, F, K, A)$  be a soft graph represented by  $\{H(x) : x \in A\}$ . Then the line graph  $L[H(x)]$  of the part  $H(x)$  of  $G$  is a graph whose vertices can be put in one-to-one correspondence with the edges of  $H(x)$  such that two vertices  $u$  and  $v$  of  $L[H(x)]$  are adjacent if and only if the corresponding edges of  $H(x)$  are adjacent.

**Definition 6.2.** [Line Graph of a Soft Graph] Let  $G = (G^*, F, K, A)$  be a soft graph represented by  $\{H(x) : x \in A\}$ . Then the line graph  $L(G)$  of the soft graph  $G$  is defined as  $L(G) = \{L[H(x)] : x \in A\}$ .

**Example 6.1.** Consider the graph  $G = (V, E)$  given in Figure 6 and its soft graph  $G = \{H(a), H(e)\}$  given in Figure 7. The line graph  $L(G)$  of the soft graph  $G$  is  $L(G) = \{L[H(a)], L[H(e)]\}$  and is given in Figure 9. The vertex corresponding to the edge  $mn$  in  $H(x)$  is given as  $u_{mn}$  in  $L[H(x)]$ .

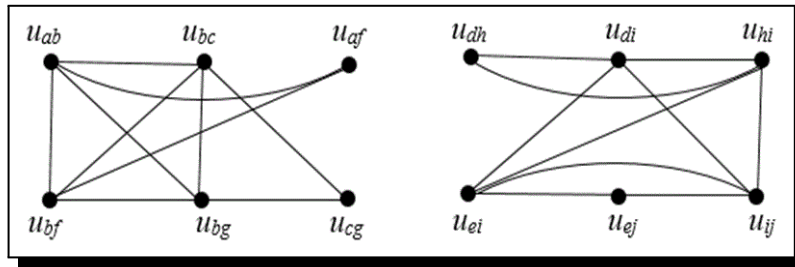


Figure 9.  $L(G) = \{L[H(a)], L[H(e)]\}$

**Theorem 6.1.** Let  $G = (G^*, F, K, A)$  be a soft graph represented by  $\{H(x) : x \in A\}$  and  $L(G)$  be the line graph of  $G$  given by  $\{L[H(x)] : x \in A\}$ . Assume that the degree sequence of the part  $H(x)$  for some  $x \in A$  is  $d_1[H(x)], d_2[H(x)] \dots d_{|F(x)|}[H(x)]$ . In  $L(G)$ , the total number of vertices is  $\sum_{x \in A} |K(x)|$  and total number of edges is  $\sum_{x \in A} \sum_{i=1}^{|F(x)|} \frac{(d_i[H(x)])(d_i[H(x)]-1)}{2}$ , where  $|F(x)|$  and  $|K(x)|$  denote the number of vertices and edges respectively in the part  $H(x)$  of  $G$ .

*Proof.* Consider a part  $H(x) = (F(x), K(x))$  of the soft graph  $G$  for some  $x \in A$  having degree sequence  $d_1[H(x)], d_2[H(x)] \dots d_{|F(x)|}[H(x)]$ . Since the vertices of the line graph  $L[H(x)]$  of this part  $H(x)$  can be put in one-to-one correspondence with the edges of  $H(x)$ , the number of vertices in  $L[H(x)]$  is  $|K(x)|$ . Also, each vertex in  $H(x)$  having degree  $d_i$  creates  $\frac{d_i(d_i-1)}{2}$  edges in  $L[H(x)]$  since there is an edge in  $L[H(x)]$  joining two vertices  $u$  and  $v$  if and only if the corresponding edges of  $H(x)$  are adjacent. So in  $L[H(x)]$ , the total number of vertices is  $|K(x)|$  and total number of edges is  $\sum_{i=1}^{|F(x)|} \frac{(d_i[H(x)])(d_i[H(x)]-1)}{2}$ . This is true for  $H(x)$  for every  $x \in A$ . So in  $L(G)$ , the total

number of vertices is  $\sum_{x \in A} |K(x)|$  and total number of edges is  $\sum_{x \in A} \sum_{i=1}^{|F(x)|} \frac{(d_i[H(x)])(d_i[H(x)]-1)}{2}$ .  $\square$

**Example 6.2.** Consider the soft graph  $G = \{H(a), H(e)\}$  given in Figure 7 and its line graph given in Figure 9.

Here the total number of vertices in  $L(G) = 12 = 6 + 6 = |K(a)| + |K(e)| = \sum_{x \in A} |K(x)|$  and the total number of edges in  $L(G) = 20 = 10 + 10 = \sum_{i=1}^{|F(a)|} \frac{(d_i[H(a)])(d_i[H(a)]-1)}{2} + \sum_{i=1}^{|F(e)|} \frac{(d_i[H(e)])(d_i[H(e)]-1)}{2} = \sum_{x \in A} \sum_{i=1}^{|F(x)|} \frac{(d_i[H(x)])(d_i[H(x)]-1)}{2}$ .

## 7. Conclusion

Theory of soft graphs is a fast developing research area in graph theory due to its capability to deal with the parameterization tool. Soft graph was introduced by applying the concept of soft set in graph. By means of parameterization, soft graph produces a series of descriptions of a complicated relation described using a graph. We introduced the concepts of subdivision graph, power and line graph of a soft graph and established some important properties of them.

## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

## References

- [1] M. Akram and S. Nawaz, Certain types of soft graphs, *UPB Scientific Bulletin, Series A: Applied Mathematics and Physics* **78**(4) (2016), 67 – 82, URL: [https://www.scientificbulletin.upb.ro/rev\\_docs\\_arhiva/fullcc2\\_842873.pdf](https://www.scientificbulletin.upb.ro/rev_docs_arhiva/fullcc2_842873.pdf).
- [2] M. Akram and S. Nawaz, Operations on soft graphs, *Fuzzy Information and Engineering* **7** (2015), 423 – 449, DOI: 10.1016/j.fiae.2015.11.003.
- [3] G. Chartrand, L. Lesniak and P. Zhang, *Graphs & Digraphs*, 6th edition, Chapman and Hall/CRC (2016), p. 640, DOI: 10.1201/b19731.
- [4] J. Clark and D.A. Holton, *A First Look at Graph Theory*, Allied Publishers Ltd. (1995), [https://inoerofik.files.wordpress.com/2014/11/firstlook\\_graphtheory.pdf](https://inoerofik.files.wordpress.com/2014/11/firstlook_graphtheory.pdf).
- [5] P.K. Maji, A.R. Roy and R. Biswas, An application of soft sets in a decision making problem, *Computers and Mathematics with Application* **44** (2002), 1077 – 1083, DOI: 10.1016/S0898-1221(02)00216-X.
- [6] P.K. Maji, A.R. Roy and R. Biswas, Fuzzy soft sets, *The Journal of Fuzzy Mathematics* **9** (2001), 589 – 602.
- [7] D. Molodtsov, Soft set theory – First results, *Computers & Mathematics with Applications* **37** (1999), 19 – 31, DOI: 10.1016/S0898-1221(99)00056-5.
- [8] N. Sarala and K. Manju, On soft bi-partite graph, *International Journal of Basic and Applied Research* **9** (2019), 249 – 256.
- [9] J.D. Thenge, B.S. Reddy and R.S. Jain, Adjacency and incidence matrix of a soft graph, *Communications in Mathematics and Applications* **11**(1) (2020), 23 – 30, DOI: 10.26713/cma.v11i1.1281.

- [10] J.D. Thenge, B.S. Reddy and R.S. Jain, Connected soft graph, *New Mathematics and Natural Computation* **16**(2) (2020), 305 – 318, DOI: 10.1142/S1793005720500180.
- [11] J.D. Thenge, B.S. Reddy and R.S. Jain, Contribution to soft graph and soft tree, *New Mathematics and Natural Computation* **15**(1) (2019), 129 – 143, DOI: 10.1142/S179300571950008X.
- [12] R.K. Thumbakara and B. George, Soft graphs, *General Mathematics Notes* **21**(2) (2014), 75 – 86, URL: [https://www.emis.de/journals/GMN/yahoo\\_site\\_admin/assets/docs/6\\_GMN-4802-V21N2.16902935.pdf](https://www.emis.de/journals/GMN/yahoo_site_admin/assets/docs/6_GMN-4802-V21N2.16902935.pdf).
- [13] S. Venkatraman and R. Helen, On domination in soft graph of some special graphs, *Malaya Journal of Matematik* **S**(1) (2019), 527 – 531, URL: <https://malayajournal.org/download.php?id=709>.