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An Introduction to Soft Hypergraphs

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Abstract

This is an introductory paper on Soft Hypergraph. In 1999, D. Molodtsov initiated the concept of soft set theory. This is an approach for modeling vagueness and uncertainty. The concept of soft graphs introduced by Rajesh K. Thumbakara and Bobin George is used to provide a parameterized point of view for graphs. Theory of soft graphs is a fast developing area in graph theory due to its capability to deal with the parameterization tool. Hypergraph is a generalization of graph. In this paper we introduce the concept of soft hypergraph by applying the idea of soft set in hypergraph. By means of parameterization, soft hypergraph produces a series of descriptions of a relation described using a hypergraph.

Keywords: Soft Set, Soft Graph, Soft Hypergraph.
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1. Introduction

Hypergraph is a generalization of the graph in which any subset of the vertex set can be an edge . Hypergraphs were introduced by Berge [3] in 1973. The idea of soft sets was first given by D. Molodtsov [8] in 1999. This is a new mathematical tool to deal with the uncertainties. Many practical problems can be solved easily with the help of soft set theory rather than some well-known theories viz. fuzzy set theory, probability theory etc. since these theories have certain limitations. The problem with the fuzzy set is that it lacks parameterization tools. Many authors like P.K. Maji, A.R. Roy and R. Biswas [6], [7] have further studied the theory of soft sets and used the theory to solve some decision making problems. In 2014, Rajesh K. Thumbakara and Bobin George [13],[14] introduced the concept of soft graph to provide a parameterized point of view for graphs. In 2015, Muhammad Akram and Saira Nawas [1] modified the definition of soft graph. Further studies on soft graphs were conducted by J.D. Thenge, B.S. Reddy, R.S. Jain [10], [11], [12], N.Sarala, K. Manju [9] and S. Venkatraman, R. Helen [15]. In this paper we introduce the concept of soft hypergraph.

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2. Preliminaries

2.1. Hypergraph

For basic concepts of hypergraph we refer [3] , [4] and [16]. Let $V = \{v_1, v_2, \ldots, v_n\}$ be a finite set, and let $E = \{E_1, E_2, \ldots, E_m\}$ be a family of subsets of V. The pair $H = (V, E)$ is called a *hypergraph* with vertex set V and edge set E. The vertex set V and the edge set E are also denoted by $V(H)$ and $E(H)$ respectively. Here $|V| = n$ is called the *order* of the hypergraph H. The order of H is also denoted by $n(H)$. The elements $v_1, v_2, \ldots, v_n$ are called *vertices* of the hypergraph and the sets $E_1, E_2, \ldots, E_m$ are called *hyperedges* (or simply *edges*) of the hypergraph H. Some edges of a hypergraph may be empty sets. The number of hyperedges in H is called the *size* of H and is also denoted by $m(H)$. Some edges may be the subsets of other edges; in this case they are called *included*. If some hyperedges coincide, they are called *multiple*. If a hypergraph contains no included edges, it is called a *simple hypergraph*. Hence a simple hypergraph do not have empty and multiple edges. Two vertices in a hypergraph H are said to be *adjacent* if there is a hyperedge in H that contains both vertices. The adjacent vertices are called *neighbor* to each other. The *neighborhood* of a given vertex v of a hypergraph is the set of all neighbors of v and is denoted by $N(v)$. Two hyperedges in a hypergraph are said to be *adjacent* if their intersection is not empty. If a vertex v belongs to a hyperedge E_j, then we say that they are *incident* to each other. If $E(v), v \in V$ denotes all the edges containing the vertex v, the number $|E(v)|$ is called the *degree* of the vertex v. The number $|E_i|$ is called *size* or *cardinality* of the edge E_i. An edge of a hypergraph which contains no vertices is called an *empty edge*. The size of an empty edge is trivially 0. A vertex of a hypergraph which is incident to no edges is called an *isolated vertex*. The degree of an isolated vertex is trivially 0. An edge having cardinality 1 is called a *loop*. A vertex of degree 1 is called a *pendent vertex*. If in a hypergraph, each vertex has degree $k > 0$, it is called a *k-regular*. If each hyperedge in a hypergraph has the same cardinality $r > 0$, then it is called *r-uniform*.

2.2. Soft Set

In 1999 D. Molodtsov [8] initiated the concept of soft sets. Let U be an initial universe set and let E be a set of parameters. A pair (F, E) is called a Soft Set (over U) if and only F is a mapping of E into the set of all subsets of the set U. That is, $F : E \to P(U)$.

3. Soft Hypergraphs

Definition 3.1. Let $H = (V, E)$ be a hypergraph with vertex set V and hyperedge (edge) set E. Then $D_i \subseteq V$ is called a *subhyperedge* of a hyperedge E_j in H if $D_i \subseteq E_j$. We also say that D_i is a subhyperedge of H. Clearly a hyperedge is a subhyperedge of itself. If $D_i \subset E_j$ then the subhyperedge D_i is said to be proper.

Definition 3.2. Let $H = (V, E)$ be a hypergraph. Any hypergraph $H' = (V', E')$ is called a *semisubhypergraph* of H if $V' \subseteq V$ and each element in E' is a subhyperedge of a hyperedge in H.

Definition 3.3. Let $H = (V, E)$ be a hypergraph and $V' \subseteq V$. Then $D_i \subseteq V'$ is said to be a *maximum subhyperedge* or *m-subhyperedge*, if D_i is a subhyperedge of some hyperedge E_j in H and is not a proper subhyperedge of any other subhyperedge of H that can be formed using vertices of V'. The set of all maximum subhyperedges of H that can be formed from the vertices in V' is denoted by {m-subhyperedges $\langle V' \rangle$}.

Definition 3.4. Let $H^* = (V, E)$ be a simple hypergraph with vertex set V and edge(hyperedge) set E and C be any nonempty set. let E_s be the set of all subhyperedges of H^*. Let R be an arbitrary relation between elements of C and elements of V. That is $R \subseteq C \times V$. A mapping $A : C \to P(V)$ can be defined as $A(c) = \{v \in V : cRv\}$. Also define a mapping $B : C \to P(E_s)$ by $B(c)$={m-subhyperedges $\langle A(c) \rangle$}. The pair (A, C) is a soft set over V and the pair (B, C) is a soft set over E_s. Then the 4 -tuple $H = (H^*, A, B, C)$ is called a *Soft Hypergraph* if it satisfies the following conditions:

1. $H^* = (V, E)$ is a simple hypergraph,
2. C is a nonempty set of parameters,
3. (A, C) is a soft set over V,
4. (B, C) is a soft set over E_s,
5. $(A(c), B(c))$ is a semisubhypergraph of H^* for all $c \in C$.

If we represent $(A(c), B(c))$ by $F(c)$ then the soft hypergraph H is also given by $\{F(c) : c \in C\}$. Then $F(c)$ corresponding to a parameter c in C is called a *hyperpart* or simply *h-part* of the soft hypergraph H.

Example 3.1. Consider a hypergraph $H^* = (V, E)$ given in Fig. 1. Let $C = \{v_2, v_7\} \subseteq V$ be a parameter

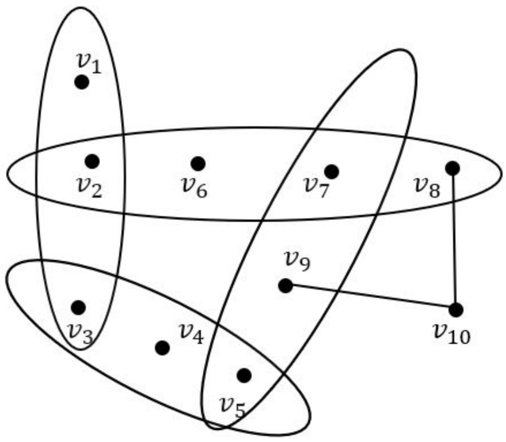

Figure 1: Hypergraph $H^* = (V, E)$

set. Define a function $A : C \to P(V)$ defined by $A(c) = \{v \in V : cRv \Leftrightarrow c = v \text{ or } c \text{ and } v \text{ are adjacent}\}$ for all $c \in C$. That is, $A(v_2) = \{v_1, v_2, v_3, v_6, v_7, v_8\}$ and $A(v_7) = \{v_2, v_5, v_6, v_7, v_8, v_9\}$. Then (A, C) is a soft set over V. Define another function $B : C \to P(E_s)$ defined by $B(c) = \{\text{m-subhyperedges } \langle A(c)\rangle\}$ for all $c \in C$. That is, $B(v_2) = \{\{v_1, v_2, v_3\}, \{v_2, v_6, v_7, v_8\}\}$ and $B(v_7) = \{\{v_2, v_6, v_7, v_8\}, \{v_5, v_7, v_9\}\}$. Then (B, C) is a soft set over E_s. Also $F(v_2) = (A(v_2), B(v_2))$ and $F(v_7) = (A(v_7), B(v_7))$ are semisubhypergraphs of H^* as shown in Fig. 2. Hence $H = \{F(v_2), F(v_7)\}$ is a soft hypergraph of H^*.

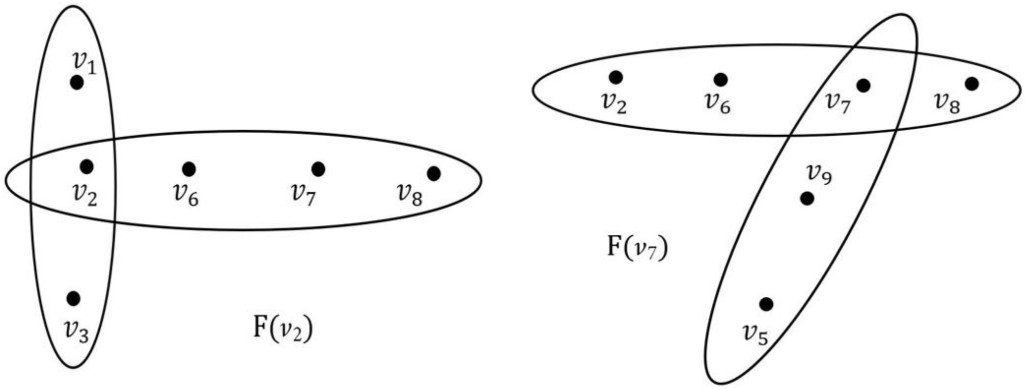

Figure 2: Soft Hypergraph $H = \{F(v_2), F(v_7)\}$

Definition 3.5. Let $H^* = (V, E)$ be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^*. Let $F(c) = (A(c), B(c))$ be an h-part of H for some $c \in C$ having $|A(c)|$ vertices and $|B(c)|$ hyperedges. Then the *order* of the soft hypergraph H, denoted by $n(H)$, is given by $n(H) = \sum_{c\in C} |A(c)|$.

Definition 3.6. Let $H = (H^*, A, B, C)$ be a soft hypergraph of a simple hypergraph H^*. Then the *size* or *cardinality* of the soft hypergraph H, denoted by $m(H)$, is given by $m(H) = \sum_{c\in C} |B(c)|$.

4. Soft Degree in Soft Hypergraph

Definition 4.1. Let $H^* = (V, E)$ be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$ where $F(c) = (A(c), B(c)), \forall c \in C$. Let v be any vertex of an h-part $F(c)$ of H for some $c \in C$. Then the *h-part degree* of the vertex v in $F(c)$, denoted by $d(v)[F(c)]$ is the degree of the vertex v in $F(c)$. That is, $d(v)[F(c)]$ is the number of hyperedges in the h-part $F(c)$ containing the vertex v. We define h-part degree in $F(c)$ only for vertices contained in $F(c)$.

Definition 4.2. Let v be any vertex of the soft hypergraph $H = (H^*, A, B, C)$. That is, $v \in \bigcup_{c\in C} A(c)$. Then the *soft degree* of the vertex v, denoted by $d(v)$ is defined by $d(v) = max\{d(v)[F(c)] : v \in A(c)$ and $c \in C\}$ where $d(v)[F(c)]$ denotes the h-part degree of the vertex v in $F(c)$.

Example 4.1. Consider a hypergraph $H^* = (V, E)$ given in Fig. 3.

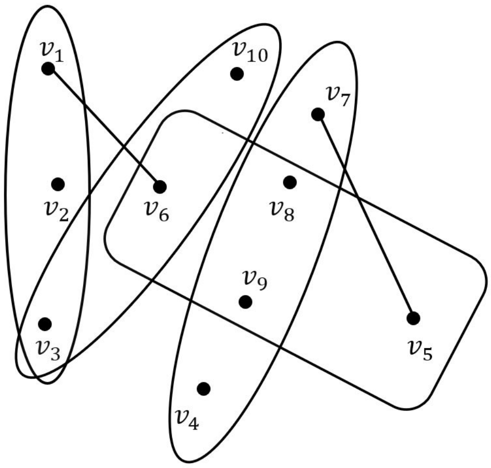

Figure 3: Hypergraph $H^* = (V, E)$

Let $C = \{v_3, v_7\} \subseteq V$ be a parameter set. Define a function $A : C \to P(V)$ defined by $A(c) = \{v \in V : cRv \Leftrightarrow c = v$ or c and v are adjacent$\}$ for all $c \in C$. That is, $A(v_3) = \{v_1, v_2, v_3, v_6, v_{10}\}$ and $A(v_7) = \{v_4, v_5, v_7, v_8, v_9\}$. Then (A, C) is a soft set over V. Define another function $B : C \to P(E_s)$ defined by $B(c) = \{$m-subhyperedges $\langle A(c)\rangle\}$ for all $c \in C$. That is, $B(v_3) = \{\{v_1, v_2, v_3\}, \{v_3, v_6, v_{10}\}, \{v_1, v_6\}\}$ and $B(v_7) = \{\{v_4, v_9, v_7, v_8\}, \{v_5, v_8, v_9\}, \{v_5, v_7\}\}$. Then (B, C) is a soft set over E_s. Also $F(v_3) = (A(v_3), B(v_3))$ and $F(v_7) = (A(v_7), B(v_7))$ are semisubhypergraphs of H^* as shown in Fig. 4. Hence $H = \{F(v_3), F(v_7)\}$ is a soft hypergraph of H^*.
Here $\bigcup_{c\in C} A(c) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$.
We have $d(v_1)[F(v_3)] = 2, d(v_2)[F(v_3)] = 1, d(v_3)[F(v_3)] = 2, d(v_6)[F(v_3)] = 2, d(v_{10})[F(v_3)] = 1$. Also we have $d(v_4)[F(v_7)] = 1, d(v_9)[F(v_7)] = 2, d(v_8)[F(v_7)] = 2, d(v_7)[F(v_7)] = 2, d(v_5)[F(v_7)] = 2$. We know $d(v) = max\{d(v)[F(c)] : v \in A(c)$ and $c \in C\}$. So $d(v_1) = max\{2\} = 2, d(v_2) = max\{1\} = 1, d(v_3) = max\{2\} = 2, d(v_4) = max\{1\} = 1, d(v_5) = max\{2\} = 2, d(v_6) = max\{2\} = 2, d(v_7) = max\{2\} = 2, d(v_8) = max\{2\} = 2, d(v_9) = max\{2\} = 2$ and $d(v_{10}) = max\{1\} = 1$.

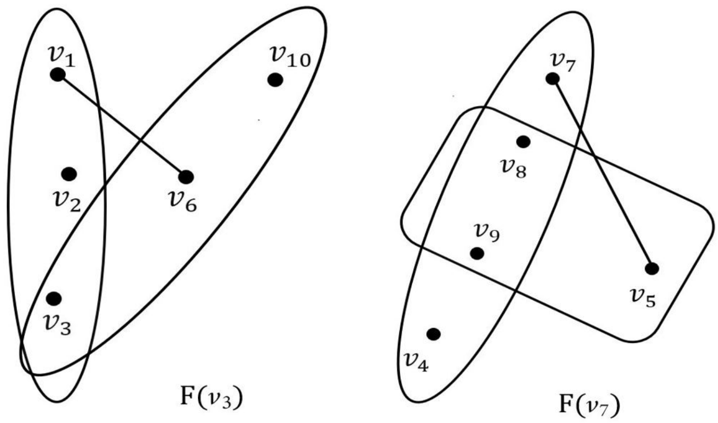

Figure 4: Soft Hypergraph $H = \{F(v_3), F(v_7)\}$

5. Soft Incidence Matrix and Soft Adjacency matrix of a Soft Hypergraph

Definition 5.1. Let $H^* = (V, E)$ be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$ where $F(c) = (A(c), B(c)), \forall c \in C$. Let $F(c)$ be an h-part of H for some $c \in C$. Then *incidence matrix* of the h-part $F(c)$, denoted by $I[F(c)]$ is a matrix of order $|A(c)| \times |B(c)|$ given by $I[F(c)] = [I_{ij}]$ where

$$I_{ij} = \begin{cases} 1, & \text{when } v_i \in E_j \text{ in } F(c) \\ 0, & \text{when } v_i \notin E_j \text{ in } F(c). \end{cases}$$

.

Definition 5.2. Let $H = (H^*, A, B, C)$ be a soft hypergraph represented by $\{F(c) : c \in C\}$. Then the *soft incidence matrix* of the soft hypergraph H is given by $I[H] = \{I[F(c)] : c \in C\}$, where $I[F(c)]$ denotes the incidence matrix of the h-part $F(c)$.

Example 5.1. Consider the soft hypergraph $H = \{F(v_3), F(v_7)\}$ given in Fig. 4 having two h-parts $F(v_3)$ and $F(v_7)$. We name the hyperedges of H as shown in Fig. 5.

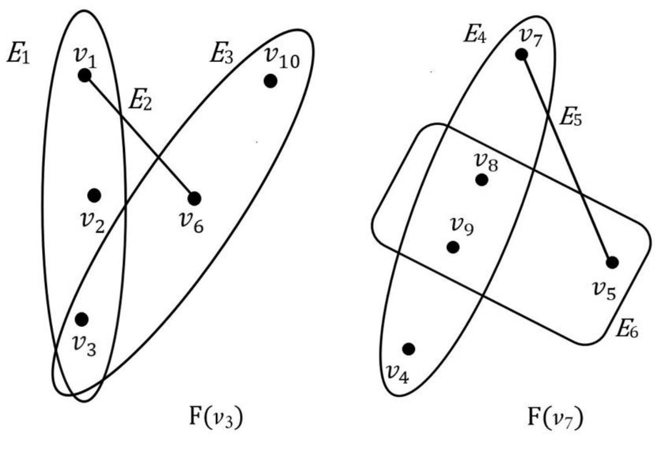

Figure 5: Soft Hypergraph $H = \{F(v_3), F(v_7)\}$

In the h-part $F(v_3)$, $A(v_3) = \{v_1, v_2, v_3, v_6, v_{10}\}$ and $B(v_3) = \{E_1, E_2, E_3\}$. So the incidence matrix of the h-part $F(v_3)$ is a 5×3 matrix given by

$$I[F(v_3)] = \begin{array}{c c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} & \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_6 \\ v_{10} \end{array} \end{array}.$$

In the h-part $F(v_7)$, $A(v_7) = \{v_4, v_5, v_7, v_8, v_9\}$ and $B(v_7) = \{E_4, E_5, E_6\}$. So the incidence matrix of the h-part $F(v_7)$ is a 5×3 matrix given by

$$I[F(v_7)] = \begin{array}{c c} \begin{array}{ccc} E_4 & E_5 & E_6 \end{array} & \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} & \begin{array}{c} v_4 \\ v_5 \\ v_7 \\ v_8 \\ v_9 \end{array} \end{array}.$$

Then soft incidence matrix of the soft hypergraph H is given by $I[H] = \{I[F(v_3)], I[F(v_7)]\}$.

Remark 5.2. We can also observe the following properties of the incidence matrix $I[F(c)]$ of an h-part $F(c)$ of a soft hypergraph H.

1. The matrix $I[F(c)]$ contains only 0 and 1 as elements,
2. If $A(c)$ contains m vertices and $B(c)$ contains n hyperedges, then the incidence matrix $I[F(c)]$ of the corresponding h-part $F(c)$ will be an $m \times n$ matrix,
3. In $I[F(c)]$, rows correspond to vertices and columns correspond to hyperedges in the h-part $F(c)$,
4. Sum of entries in a row of $I[F(c)]$ is equal to the h-part degree of the corresponding vertex in the h-part $F(c)$,
5. Sum of entries in a column of $I[F(c)]$ is equal to the cardinality or size of the corresponding hyperedge in the h-part $F(c)$.

Theorem 5.3. *Let $H^* = (V, E)$ be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$. Let $v_i, i = 1, 2, \cdots, |A(c)|$ be the vertices and $E_j, j = 1, 2, \cdots, |B(c)|$ be the hyperedges of an h-part $F(c)$ for some $c \in C$. Then*

$$\sum_{c \in C} \sum_{i=1}^{|A(c)|} d(v_i)[F(c)] = \sum_{c \in C} \sum_{j=1}^{|B(c)|} |E_j|.$$

Proof. Let $F(c)$ be any h-part of the soft hypergraph H for some $c \in C$ and $I[F(c)]$ be the incidence matrix of $F(c)$. If we sum the entries of $I[F(c)]$ by columns and by rows, we get the same number. The sum of enries of $I[F(c)]$ by rows gives the sum of h-part degrees of vertices in $F(c)$ and the sum of entries of $I[F(c)]$ by columns gives the sum of cardinalities of hyperedges in $F(c)$. Since these two sums are equal, we get

$$\sum_{i=1}^{|A(c)|} d(v_i)[F(c)] = \sum_{j=1}^{|B(c)|} |E_j|.$$

This result is true for all h-parts $F(c)$ of H. Therefore we have

$$\sum_{c\in C}\sum_{i=1}^{|A(c)|} d(v_i)[F(c)] = \sum_{c\in C}\sum_{j=1}^{|B(c)|} |E_j|.$$

□

Example 5.4. Consider the soft hypergraph $H = \{F(v_3), F(v_7)\}$ given in Fig. 5.
Here

$$\sum_{c\in C}\sum_{i=1}^{|A(c)|} d(v_i)[F(c)] = \sum_{i=1}^{|A(v_3)|} d(v_i)[F(v_3)] + \sum_{i=1}^{|A(v_7)|} d(v_i)[F(v_7)]$$

= (2+1+2+2+1) + (1+2+2+2+2) = 17.
Also

$$\sum_{c\in C}\sum_{j=1}^{|B(c)|} |E_j| = \sum_{j=1}^{|B(v_3)|} |E_j| + \sum_{j=1}^{|B(v_7)|} |E_j|$$

= (3+2+3) + (4+3+2) = 17.
That is,

$$\sum_{c\in C}\sum_{i=1}^{|A(c)|} d(v_i)[F(c)] = \sum_{c\in C}\sum_{j=1}^{|B(c)|} |E_j|.$$

Definition 5.3. Let $H^* = (V, E)$ be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$ where $F(c) = (A(c), B(c)), \forall c \in C$. Let $F(c)$ be an h-part of H for some $c \in C$ and v be any vertex of $F(c)$. Also let $E(v)$ be set of all the hyperedges in $F(c)$ containing v. Then *adjacency matrix* of the h-part $F(c)$, denoted by $D[F(c)]$ is a square matrix of order $|A(c)|$ given by $D[F(c)] = [D_{ij}]$ where

$$D_{ij} = \begin{cases} 1, \text{ if } E(v_i) \cap E(v_j) \neq \phi \text{ in } F(c) \\ 0, \text{ if } E(v_i) \cap E(v_j) = \phi \text{ in } F(c). \end{cases}$$

Definition 5.4. Let $H = (H^*, A, B, C)$ be a soft hypergraph represented by $\{F(c) : c \in C\}$. Then the *soft adjacency matrix* of the soft hypergraph H is given by $D[H] = \{D[F(c)] : c \in C\}$,where $D[F(c)]$ denotes the adjacency matrix of the h-part $F(c)$.

Example 5.5. Consider the soft hypergraph $H = \{F(v_3), F(v_7)\}$ given in Fig. 5. In the h-part $F(v_3)$, $A(v_3) = \{v_1, v_2, v_3, v_6, v_{10}\}$. The adjacency matrix of the h-part $F(v_3)$ is a square matrix of order 5 given by

$$D[F(v_3)] = \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_6 & v_{10} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_6 \\ v_{10} \end{array}.$$

In the h-part $F(v_7)$, $A(v_7) = \{v_1, v_2, v_3, v_6, v_{10}\}$. The adjacency matrix of the h-part $F(v_7)$ is a square matrix of order 5 given by

$$D[F(v_7)] = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccccc} v_4 & v_5 & v_7 & v_8 & v_9 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ v_4 \\ v_5 \\ v_7 \\ v_8 \\ v_9 \end{array}.$$

Then soft adjacency matrix of the soft hypergraph H is given by $D[H] = \{D[F(v_3)], D[F(v_7)]\}$.

Remark 5.6. We can also observe the following properties of the adjacency matrix $D[F(c)]$ of an h-part $F(c)$ of a soft hypergraph H.

1. The matrix $D[F(c)]$ contains only 0 and 1 as elements,
2. If $A(c)$ contains n vertices , then the adjacency matrix $D[F(c)]$ of the corresponding h-part $F(c)$ will be a square matrix of order n ,
3. $D[F(c)]$ will be a symmetric matrix,
4. We cannot draw a soft hypergraph in a unique way by knowing the adjacency matrices of its h-parts.

6. Regular and Uniform Soft Hypergraphs

Definition 6.1. Let H^* be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$. Let $F(c)$ be an h-part of H for some $c \in C$. Then $F(c)$ is called a *k-regular h-part* where k is a positive integer, if $d(v)[F(c)] = k, \ \forall v \in A(c)$.

Definition 6.2. A soft hypergraph $H = (H^*, A, B, C)$ is called *k-regular soft hypergraph* where k is a positive integer, if its h-part $F(c)$ is k-regular $\forall c \in C$.

Definition 6.3. A soft hypergraph $H = (H^*, A, B, C)$ is called a *regular soft hypergraph*, if its h-part $F(c)$ is k-regular for any integer $k > 0, \ \forall c \in C$.

Example 6.1. Consider a hypergraph $H^* = (V, E)$ given in Fig. 6.

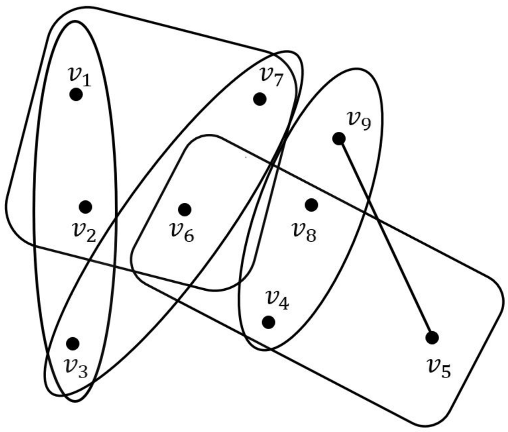

Figure 6: Hypergraph $H^* = (V, E)$

Let $C = \{v_3, v_9\} \subseteq V$ be a parameter set. Define a function $A : C \to P(V)$ defined by $A(c) = \{v \in V : cRv \Leftrightarrow c = v$ or c and v are adjacent$\}$ for all $c \in C$. That is, $A(v_3) = \{v_1, v_2, v_3, v_6, v_7\}$ and $A(v_9) =$

$\{v_4, v_5, v_8, v_9\}$. Then (A, C) is a soft set over V. Define another function $B : C \to P(E_s)$ defined by $B(c)$ $= \{$m-subhyperedges $\langle A(c)\rangle\}$ for all $c \in C$. That is, $B(v_3) = \{\{v_1, v_2, v_3\}, \{v_3, v_6, v_7\}, \{v_1, v_2, v_6, v_7\}\}$ and $B(v_9) = \{\{v_4, v_8, v_9\}, \{v_4, v_8, v_5\}, \{v_5, v_9\}\}$. Then (B, C) is a soft set over E_s. Also $F(v_3) = (A(v_3), B(v_3))$ and $F(v_9) = (A(v_9), B(v_9))$ are semisubhypergraphs of H^* as shown in Fig. 7. Hence $H = \{F(v_3), F(v_9)\}$ is a soft hypergraph of H^*. Here $d(v)[F(v_3)] = 2,\ \forall v \in A(v_3)$ and $d(v)[F(v_9)] = 2,\ \forall v \in A(v_9)$. So $F(v_3)$

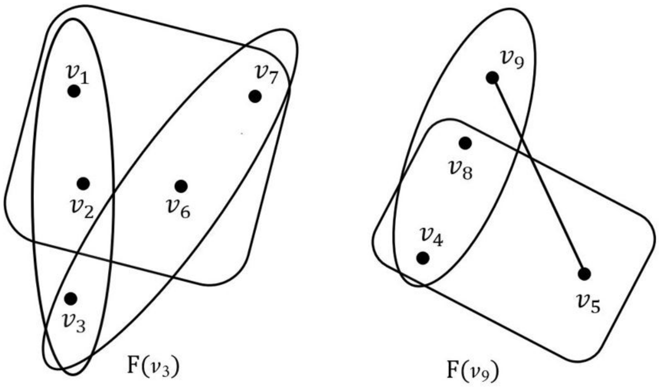

Figure 7: Soft Hypergraph $H = \{F(v_3), F(v_9)\}$

and $F(v_7)$ are 2-regular h-parts of H. Therefore H is a 2-regular soft hypergraph.

Definition 6.4. Let H^* be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$. Let $F(c)$ be an h-part of H for some $c \in C$. Then $F(c)$ is called *r-uniform h-part* where r is a positive integer, if the cardinality of the hyperedge E in $F(c)$ is $r,\ \forall E \in B(c)$.

Definition 6.5. A soft hypergraph $H = (H^*, A, B, C)$ is called *r-uniform soft hypergraph* where r is a positive integer, if its h-part $F(c)$ is r-uniform $\forall c \in C$.

Definition 6.6. A soft hypergraph $H = (H^*, A, B, C)$ is called a *uniform soft hypergraph*, if its h-part $F(c)$ is r-uniform for any integer $r > 0,\ \forall c \in C$.

Example 6.2. Consider a hypergraph $H^* = (V, E)$ given in Fig. 8.

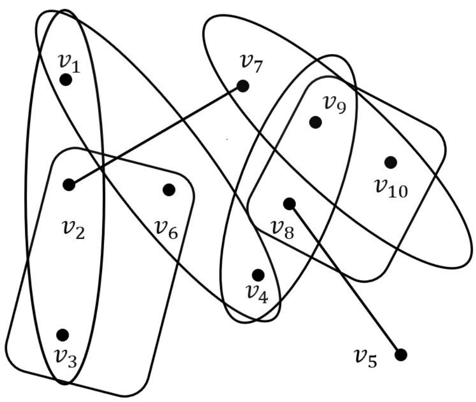

Figure 8: Hypergraph $H^* = (V, E)$

Let $C = \{v_1, v_9\} \subseteq V$ be a parameter set. Define a function $A : C \to P(V)$ defined by $A(c) = \{v \in V : cRv \Leftrightarrow c = v$ or c and v are adjacent$\}$ for all $c \in C$. That is, $A(v_1) = \{v_1, v_2, v_3, v_4, v_6\}$ and $A(v_9) = \{v_4, v_7, v_8, v_9, v_{10}\}$. Then (A, C) is a soft set over V. Define another function $B : C \to P(E_s)$ defined by $B(c) = \{\text{m-subhyperedges } \langle A(c) \rangle\}$ for all $c \in C$. That is, $B(v_1) = \{\{v_1, v_2, v_3\}, \{v_1, v_6, v_4\}, \{v_2, v_3, v_6\}\}$ and $B(v_9) = \{\{v_4, v_8, v_9\}, \{v_7, v_9, v_{10}\}, \{v_8, v_9, v_{10}\}\}$. Then (B, C) is a soft set over E_s. Also $F(v_1) = (A(v_1), B(v_1))$ and $F(v_9) = (A(v_9), B(v_9))$ are semisubhypergraphs of H^* as shown in Fig. 9. Hence $H = \{F(v_1), F(v_9)\}$ is a soft hypergraph of H^*. Here the cardinality of the hyperedge E in $F(v_1)$ is 3,

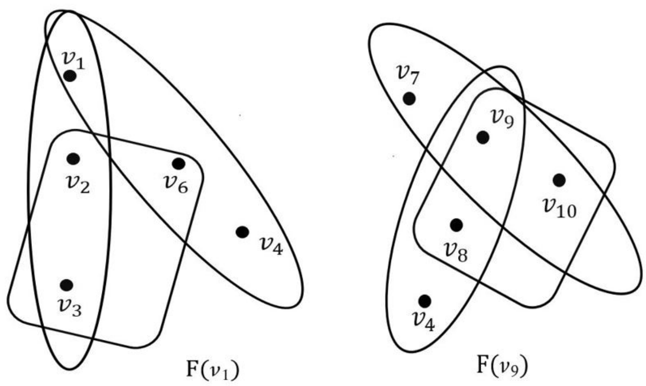

Figure 9: Soft Hypergraph $H = \{F(v_1), F(v_9)\}$

$\forall E \in B(v_1)$. Also the cardinality of the hyperedge E in $F(v_9)$ is 3, $\forall E \in B(v_9)$. That is, $F(v_1)$ and $F(v_9)$ are 3-uniform h-parts. Hence H is a 3-uniform soft hypergraph.

7. Complete r-uniform Soft Hypergraphs

Definition 7.1. Let H^* be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$. Let $F(c)$ be an h-part of H for some $c \in C$. Then $F(c)$ is called a *complete r-uniform h-part* where r is a positive integer and $0 \leq r \leq |A(c)|$, if $B(c)$ coincides with all the r-subsets of $A(c)$.

Definition 7.2. A soft hypergraph $H = (H^*, A, B, C)$ is called *complete r-uniform soft hypergraph* where r is a positive integer, if its h-part $F(c)$ is complete r-uniform $\forall c \in C$. If H has k h-parts $F(c_1), F(c_2), \cdots, F(c_k)$ having $n_1, n_2, \ldots, n_k$ vertices then the complete r-uniform soft hypergraph H is represented by $K^r_{n_1,n_2,\ldots,n_k}$.

Definition 7.3. A soft hypergraph $H = (H^*, A, B, C)$ is called a *complete uniform soft hypergraph*, if its h-part $F(c)$ is complete r-uniform for any integer $r > 0$, $\forall c \in C$. If H has k h-parts $F(c_1), F(c_2), \cdots, F(c_k)$ having $n_1, n_2, \ldots, n_k$ vertices and are resectively r_1-uniform, r_2-uniform, $\ldots$, r_k-uniform, then the complete uniform soft hypergraph H is represented by $K^{r_1,r_2,\ldots,r_k}_{n_1,n_2,\ldots,n_k}$.

Example 7.1. Consider a hypergraph $H^* = (V, E)$ given in Fig. 10.

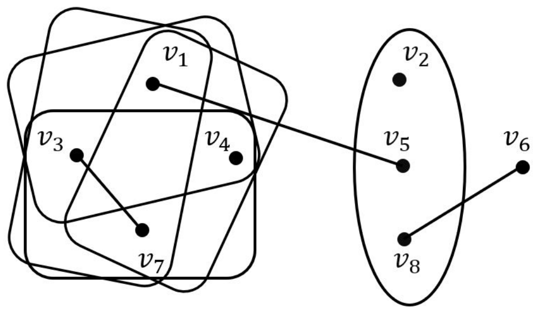

Figure 10: Hypergraph $H^* = (V, E)$

Let $C = \{v_1, v_2\} \subseteq V$ be a parameter set. Define a function $A : C \to P(V)$ defined by $A(c) = \{v \in V : cRv \Leftrightarrow c = v$ or c and v are adjacent$\}$ for all $c \in C$. That is, $A(v_1) = \{v_1, v_3, v_4, v_7\}$ and $A(v_2) = \{v_2, v_5, v_8\}$. Then (A, C) is a soft set over V. Define another function $B : C \to P(E_s)$ defined by $B(c) = \{$m-subhyperedges $\langle A(c)\rangle\}$ for all $c \in C$. That is, $B(v_1) = \{\{v_1, v_3, v_7\}, \{v_1, v_7, v_4\}, \{v_3, v_4, v_7\}, \{v_1, v_3, v_4\}\}$ and $B(v_2) = \{\{v_2, v_5, v_8\}\}$. Then (B, C) is a soft set over E_s. Also $F(v_1) = (A(v_1), B(v_1))$ and $F(v_2) = (A(v_2), B(v_2))$ are semisubhypergraphs of H^* as shown in Fig. 11. Hence $H = \{F(v_1), F(v_2)\}$ is a soft hypergraph of H^*.

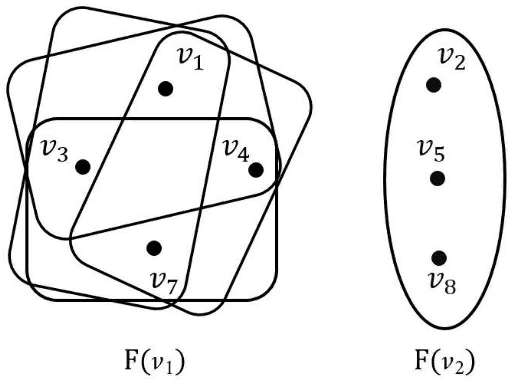

Figure 11: Soft Hypergraph $H = \{F(v_1), F(v_2)\}$

Here $B(v_1)$ coincides with all the 3-subsets of $A(v_1)$ and $B(v_2)$ coincides with all the 3-subsets of $A(v_2)$. That is, $F(v_1)$ and $F(v_2)$ are complete 3-uniform h-parts. Hence H is a complete 3-uniform soft hypergraph.

Theorem 7.2. *Let $H = (H^*, A, B, C)$ be a soft hypergraph given by $\{F(c_i) : i = 1, 2, \cdots, k\}$. If H is a complete uniform soft hypergraph in the form $K^{r_1,r_2,\ldots,r_k}_{n_1,n_2,\ldots,n_k}$, then its soft cardinality $m(H)$ is $\sum_{i=1}^{k} \frac{n_i!}{r_i!(n_i-r_i)!}$.*

Proof. Consider the complete uniform soft hypergraph $K^{r_1,r_2,\ldots,r_k}_{n_1,n_2,\ldots,n_k}$. Let $F(c_i)$ be its i^{th} h-part having n_i vertices. Clearly $A(c_i)$ has $\binom{n_i}{r_i} = \frac{n_i!}{r_i!(n_i-r_i)!}$ r_i-subsets, where $0 \leq r_i \leq |A(c_i)| = n_i$. So $B(c_i)$ contains $\frac{n_i!}{r_i!(n_i-r_i)!}$ hyperedges. That is, $|B(c_i)| = \frac{n_i!}{r_i!(n_i-r_i)!}$. Therefore the soft cardinality, $m(H) = \sum_{i=1}^{k} |B(c_i)| = \sum_{i=1}^{k} \frac{n_i!}{r_i!(n_i-r_i)!}$. □

Corollary 7.3. *If H is a complete r-uniform soft hypergraph in the form $K^{r}_{n_1,n_2,\ldots,n_k}$, then its soft cardinality $m(H)$ is $\sum_{i=1}^{k} \frac{n_i!}{r!(n_i-r)!}$.*

Proof. The result is clear from the proof of the above theorem since $r_i = r$ for the h-part $F(c_i)$, $i = 1, 2, \ldots, k$ of $K^{r}_{n_1,n_2,\ldots,n_k}$. □

8. Bipartite and Complete r-partite Soft Hypergraphs

Definition 8.1. Let H^* be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$. Let $F(c)$ be an h-part of H for some $c \in C$. Then $F(c)$ is called a *bipartite h-part* , if $A(c)$ can be partitioned into two disjoint sets $A_1(c)$ and $A_2(c)$ which are called parts in such a way that each hyperedge in $B(c)$ having cardinality ≥ 2 contains vertices from both parts. That is, there is no hyperedge having cardinality ≥ 2 in $B(c)$ which is a subset of $A_1(c)$ or $A_2(c)$.

Definition 8.2. A soft hypergraph $H = (H^*, A, B, C)$ is called *bipartite soft hypergraph*, if its h-part $F(c)$ is bipartite $\forall c \in C$.

Example 8.1. Consider the soft hypergraph $H = \{F(v_1), F(v_9)\}$ given in Fig. 9. Here $A(v_1) = \{v_1, v_2, v_3, v_4, v_6\}$ can be partitioned into two disjoint subsets $A_1(v_1) = \{v_1, v_3\}$ and $A_2(v_1) = \{v_2, v_6, v_4\}$ such that there is no hyperedge having cardinality ≥ 2 in $B(v_1)$ which is a subset of $A_1(v_1)$ or $A_2(v_1)$. Hence $F(v_1)$ is a bipartite h-part. Similarly $A(v_9) = \{v_4, v_7, v_8, v_9, v_{10}\}$ can be partitioned into two disjoint subsets $A_1(v_9) = \{v_7, v_9\}$ and $A_2(v_9) = \{v_4, v_8, v_{10}\}$ such that there is no hyperedge having cardinality ≥ 2 in $B(v_9)$ which is a subset of $A_1(v_9)$ or $A_2(v_9)$. Hence $F(v_9)$ is also a bipartite h-part. Therefore $H = \{F(v_1), F(v_9)\}$ is a bipartite soft hypergraph.

Definition 8.3. Let H^* be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$. Let $F(c)$ be an h-part of H for some $c \in C$. Then $F(c)$ is called a *complete r-partite h-part* if $F(c)$ is r-uniform and $A(c)$ can be partitioned into r non-empty parts $A_1(c), A_2(c), \cdots, A_r(c)$ in such a way that each hyperedge in $B(c)$ contains precisely one vertex from each part.

Definition 8.4. A soft hypergraph $H = (H^*, A, B, C)$ is called *complete r-partite soft hypergraph*, if its h-part $F(c)$ is complete r-partite $\forall c \in C$.

Example 8.2. Consider a hypergraph $H^* = (V, E)$ given in Fig. 12.

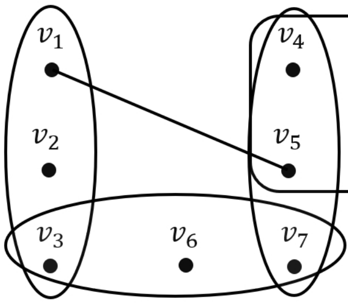

Figure 12: Hypergraph $H^* = (V, E)$

Let $C = \{v_3, v_7\} \subseteq V$ be a parameter set. Define a function $A : C \to P(V)$ defined by $A(c) = \{v \in V : cRv \Leftrightarrow c = v \text{ or } c \text{ and } v \text{ are adjacent}\}$ for all $c \in C$. That is, $A(v_3) = \{v_1, v_2, v_3, v_6, v_7\}$ and $A(v_7) = \{v_3, v_4, v_5, v_6, v_7\}$. Then (A, C) is a soft set over V. Define another function $B : C \to P(E_s)$ defined by $B(c) = \{\text{m-subhyperedges } \langle A(c)\rangle\}$ for all $c \in C$. That is, $B(v_3) = \{\{v_1, v_2, v_3\}, \{v_3, v_6, v_7\}\}$ and $B(v_7) = \{\{v_3, v_6, v_7\}, \{v_4, v_5, v_7\}\}$. Then (B, C) is a soft set over E_s. Also $F(v_3) = (A(v_3), B(v_3))$ and $F(v_7) = (A(v_7), B(v_7))$ are semisubhypergraphs of H^* as shown in Fig. 13.

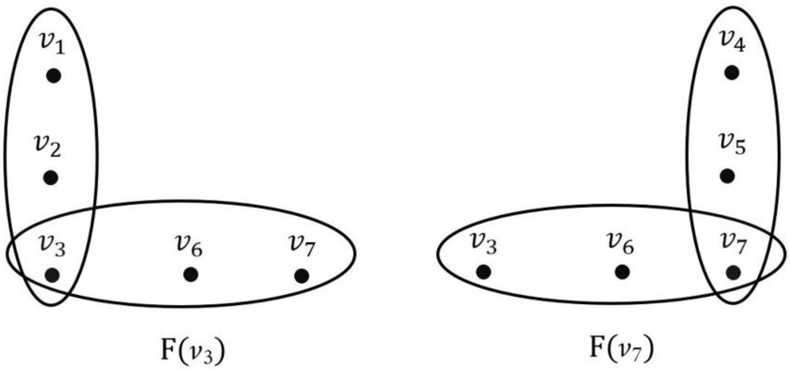

Figure 13: Soft Hypergraph $H = \{F(v_3), F(v_7)\}$

Hence $H = \{F(v_3), F(v_7)\}$ is a soft hypergraph of H^*. Here $F(v_3)$ is 3-uniform and $A(v_3) = \{v_1, v_2, v_3, v_6, v_7\}$ can be partitioned into three non-empty parts $A_1(v_3) = \{v_1, v_7\}, A_2(v_3) = \{v_2, v_6\}$ and $A_3(v_3) = \{v_3\}$ such that each hyperedge in $B(v_3)$ contains precisely one vertex from each part. Hence $F(v_3)$ is a complete r-partite h-part. Similarly $F(v_7)$ is 3-uniform and $A(v_7) = \{v_3, v_4, v_5, v_6, v_7\}$ can be partitioned into three non-empty parts $A_1(v_7) = \{v_3, v_4\}, A_2(v_7) = \{v_5, v_6\}$ and $A_3(v_7) = \{v_7\}$ such that each hyperedge in $B(v_7)$ contains precisely one vertex from each part. Hence $F(v_7)$ is also a complete r-partite h-part. Therefore $H = \{F(v_3), F(v_7)\}$ is a complete r-partite soft hypergraph.

9. Dual of a Soft Hypergraph

Definition 9.1. Let H^* be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$. Let $F(c)$ be an h-part of H for some $c \in C$ having vertex set $A(c) = \{v_1, v_2, \cdots, v_n\}$ and hyperedge set $\{E_1, E_2, \cdots, E_m\}$. Then *dual of the h-part* $F(c)$ is a hypergraph $F^*(c) = (U, D)$ whose vertex set is $U = \{e_1, e_2, \cdots, e_m\}$ and the hyperedge set is $D = \{V_1, V_2, \cdots, V_n\}$, where $V_i = \{e_j : v_i \in E_j \text{ in } F(c)\}$.

Definition 9.2. Let $H = (H^*, A, B, C)$ be a soft hypergraph represented by $\{F(c) : c \in C\}$. Then the *dual* H^* of the hypergraph H is defned as $H^* = \{F^*(c) : c \in C\}$ where $F^*(c)$ is the dual of the h-part $F(c), \forall c \in C$.

Example 9.1. Consider the soft hypergraph $H = \{F(v_3), F(v_9)\}$ given in Fig. 7. Let the hyperedges be named as given in Fig. 14.

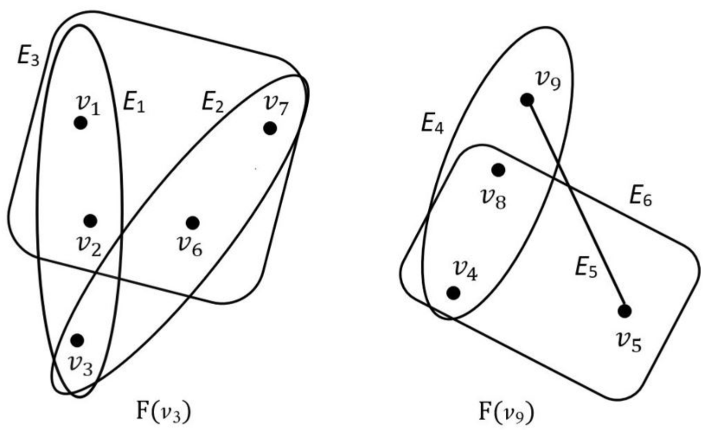

Figure 14: Soft Hypergraph $H = \{F(v_3), F(v_9)\}$

The dual of the h-part $F(v_3)$ is a hypergraph $F^*(v_3) = (U, D)$ whose vertex set is $U = \{e_1, e_2, e_3\}$ and the hyperedge set is $D = \{V_1, V_2, V_3, V_6, V_7\}$, where $V_1 = \{e_1, e_3\}, V_2 = \{e_1, e_3\}, V_3 = \{e_1, e_2\}, V_6 = \{e_2, e_3\}$ and $V_7 = \{e_2, e_3\}$. Similarly the dual of the h-part $F(v_9)$ is a hypergraph $F^*(v_9) = (W, J)$ whose vertex set is $W = \{e_4, e_5, e_6\}$ and the hyperedge set is $J = \{V_4, V_5, V_8, V_9\}$, where $V_4 = \{e_4, e_6\}, V_5 = \{e_5, e_6\}, V_8 = \{e_4, e_6\}$ and $V_9 = \{e_4, e_5\}$. Then the dual of the hypergraph $H = \{F(v_3), F(v_9)\}$ is $H^* = \{F^*(v_3), F^*(v_9)\}$ which is shown in Fig. 15.

Theorem 9.2. *Let $H = (H^*, A, B, C)$ be a soft hypergraph represented by $\{F(c) : c \in C\}$. Let $H^* = \{F^*(c) : c \in C\}$ be the dual of the soft hypergraph H. Then the soft incidence matrix of H^* is given by $I[H^*] = \{[I[F(c)]]^T : c \in C\}$ where $[I[F(c)]]^T$ denotes the transpose of the incidence matrix of the h-part $F(c)$.*

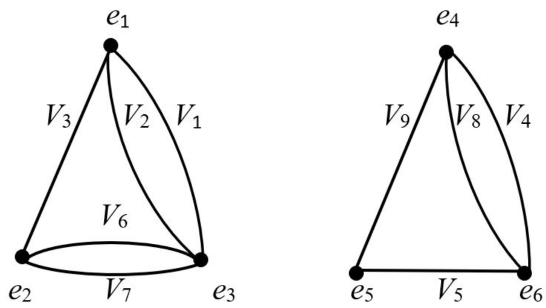

Figure 15: Dual $H^* = \{F^*(v_3), F^*(v_9)\}$

Proof. Let $F(c)$ be an h-part of the soft hypergraph H for some $c \in C$. Then incidence matrix of the h-part $F(c)$, denoted by $I[F(c)]$ is a matrix of order $|A(c)| \times |B(c)|$ given by $I[F(c)] = [I_{ij}]$ where

$$I_{ij} = \begin{cases} 1, \text{ when } v_i \in E_j \text{ in } F(c) \\ 0, \text{ when } v_i \notin E_j \text{ in } F(c). \end{cases}$$

Then the transpose of $I[F(c)]$ is given by $[I[F(c)]]^T = [I_{ij}^T]$ where

$$I_{ij}^T = \begin{cases} 1, \text{ when } v_j \in E_i \text{ in } F(c) \\ 0, \text{ when } v_j \notin E_i \text{ in } F(c). \end{cases}$$

Then by the definition of the dual of an h-part we can write $[I[F(c)]]^T = [I_{ij}^T]$ where

$$I_{ij}^T = \begin{cases} 1, \text{ when } e_i \in V_j \text{ in } F^*(c) \\ 0, \text{ when } e_i \notin V_j \text{ in } F^*(c). \end{cases}$$

The matrix $[I[F(c)]]^T$ of order $|B(c)| \times |A(c)|$ is same as the incidence matrix $I[F^*(c)]$ of the dual $F^*(c)$. Hence $I[H^*] = \{I[F^*(c)] : c \in C\} = \{[I[F(c)]]^T : c \in C\}$. □

Theorem 9.3. *Let H^* be the dual of a soft hypergraph $H = (H^*, A, B, C)$. Then the dual of H^* is H. That is, $(H^*)^* = H$.*

Proof. By the above theorem , the soft incidence matrix of H^* is given by $I[H^*] = \{[I[F(c)]]^T : c \in C\}$ where $[I[F(c)]]^T$ denotes the transpose of the incidence matrix of the h-part $F(c)$. So $I[(H^*)^*] = \{[[I[F(c)]]^T]^T : c \in C\} = \{I[F(c)] : c \in C\} = I[H]$. Since $I[(H^*)^*] = I[H]$, we have $(H^*)^* = H$. □

10. Line Graph and 2-section of a Soft Hypergraph

Definition 10.1. Let $H^* = (V, E)$ be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$. Let $F(c)$ be an h-part of H for some $c \in C$. Then the *line graph of the h-part* $F(c)$ denoted by $L[F(c)]$ is the graph with vertex set $B(c)$ and two vertices in $L[F(c)]$ are adjacent if and only if the corresponding hyperedges in $F(c)$ intersect. That is , $L[F(c)]$ is the graph $(B(c), D(c))$ having vertex set $B(c)$ and edge set $D(c)$, where the edge $(E_i, E_j) \in D(c) \Leftrightarrow E_i \cap E_j \neq \phi$ in the h-part $F(c)$.

Definition 10.2. Let $H = (H^*, A, B, C)$ be a soft hypergraph represented by $\{F(c) : c \in C\}$. Then the *line graph of the soft hypergraph* H denoted by $L(H)$ is given by $L(H) = \{L[F(c)] : c \in C\}$ where $L[F(c)]$ denotes the line graph of the h-part $F(c)$.

Definition 10.3. Let $H^* = (V, E)$ be a simple hypergraph and $H = (H^*, A, B, C)$ be a soft hypergraph of H^* represented by $\{F(c) : c \in C\}$. Let $F(c)$ be an h-part of H for some $c \in C$. Then the *2-section of the h-part* $F(c)$ denoted by $[F(c)]_2$ is the graph with the same vertex set $A(c)$, and two vertices in $[F(c)]_2$ are adjacent if and only if they both belong to a hyperedge in $F(c)$. That is, $[F(c)]_2$ is the graph $(A(c), D(c))$ having vertex set $A(c)$ and edge set $D(c)$ and the edge $(v_i, v_j) \in D(c) \Leftrightarrow E(v_i)[F(c)] \cap E(v_j)[F(c)] \neq \phi$ where $E(v)[F(c)]$ denotes the set of all hyperedges in $F(c)$ containing the vertex v.

Definition 10.4. Let $H = (H^*, A, B, C)$ be a soft hypergraph represented by $\{F(c) : c \in C\}$. Then the *2-section of the soft hypergraph* H denoted by $(H)_2$ is given by $(H)_2 = \{[F(c)]_2 : c \in C\}$ where $[F(c)]_2$ denotes the 2-section of the h-part $F(c)$.

Example 10.1. Consider a hypergraph $H^* = (V, E)$ given in Fig. 16.

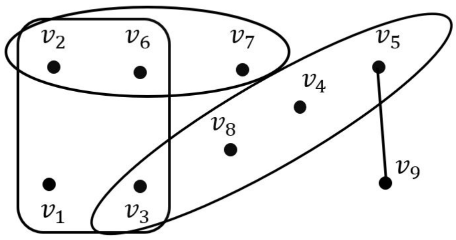

Figure 16: Hypergraph $H^* = (V, E)$

Let $C = \{v_2, v_5\} \subseteq V$ be a parameter set. Define a function $A : C \to P(V)$ defined by $A(c) = \{v \in V : cRv \Leftrightarrow c = v \text{ or } c \text{ and } v \text{ are adjacent}\}$ for all $c \in C$. That is, $A(v_2) = \{v_1, v_2, v_3, v_6, v_7\}$ and $A(v_5) = \{v_3, v_4, v_5, v_8, v_9\}$. Then (A, C) is a soft set over V. Define another function $B : C \to P(E_s)$ defined by $B(c) = \{\text{m-subhyperedges } \langle A(c)\rangle\}$ for all $c \in C$. That is, $B(v_2) = \{\{v_2, v_6, v_7\}, \{v_2, v_6, v_1, v_3\}\}$ and $B(v_5) = \{\{v_3, v_8, v_4, v_5\}, \{v_5, v_9\}\}$. Then (B, C) is a soft set over E_s. Also $F(v_2) = (A(v_2), B(v_2))$ and $F(v_5) = (A(v_5), B(v_5))$ are semisubhypergraphs of H^* as shown in Fig. 17. Hence $H = \{F(v_2), F(v_5)\}$ is a soft hypergraph of H^*. We name the hyperedges of H as in the Fig. 17.

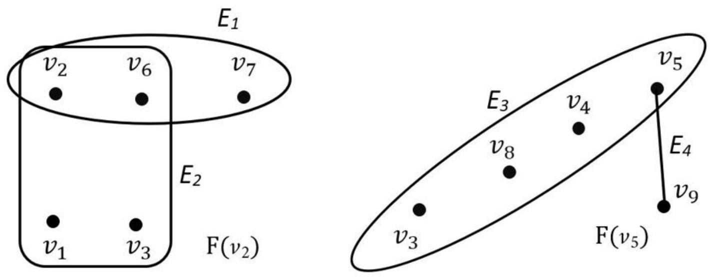

Figure 17: Soft Hypergraph $H = \{F(v_2), F(v_5)\}$

The line graph of the h-part $F(v_2)$ is the graph $L[F(v_2)] = (B(v_2), D(v_2))$ where the vertex set $B(v_2) = \{E_1, E_2\}$ and the edge set $D(v_2) = \{(E_1, E_2)\}$. Similarly the line graph of the h-part $F(v_5)$ is the graph $L[F(v_5)] = (B(v_5), D(v_5))$ where the vertex set $B(v_5) = \{E_3, E_4\}$ and the edge set $D(v_5) = \{(E_3, E_4)\}$. Then the line graph of H is given by $L(H) = \{L[F(v_2)], L[F(v_5)]\}$ and is shown in Fig. 18.

The 2-section of the h-part $F(v_2)$ is the graph $[F(v_2)]_2 = (A(v_2), D(v_2))$ where the vertex set $A(v_2) = \{v_1, v_2, v_3, v_6, v_7\}$ and the edge set $D(v_2) = \{(v_1, v_2), (v_1, v_3), (v_1, v_6), (v_2, v_3), (v_2, v_6), (v_2, v_7), (v_3, v_6), (v_6, v_7)\}$. Similarly the 2-section of the h-part $F(v_5)$ is the graph $[F(v_5)]_2 = (A(v_5), D(v_5))$ where the vertex set

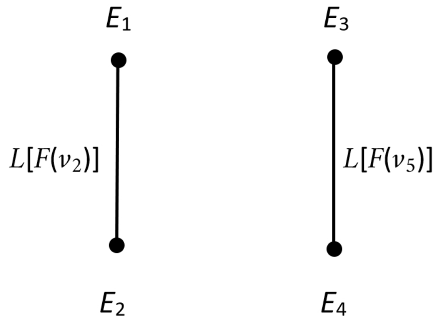

Figure 18: $L(H) = \{L[F(v_2)], L[F(v_5)]\}$

$A(v_5) = \{v_3, v_4, v_5, v_8, v_9\}$ and the edge set $D(v_5) = \{(v_3, v_4), (v_3, v_5), (v_3, v_8), (v_4, v_5), (v_4, v_8), (v_5, v_8), (v_5, v_9)\}$. Then the 2-section of H is given by $(H)_2 = \{[F(v_2)]_2, [F(v_5)]_2\}$ and is shown in Fig. 19.

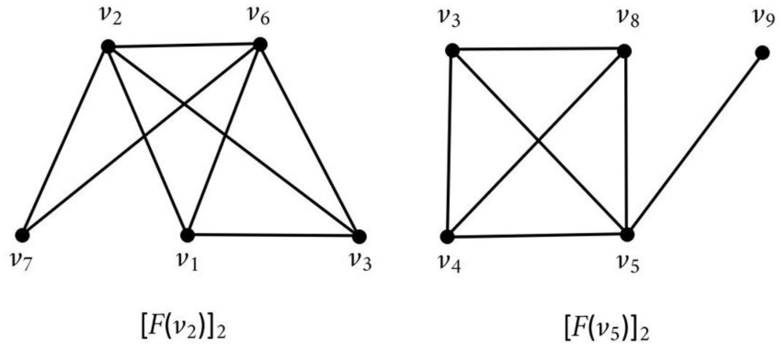

Figure 19: $(H)_2 = \{[F(v_2)]_2, [F(v_5)]_2\}$

Theorem 10.2. *Let $H = (H^*, A, B, C)$ be a soft hypergraph represented by $\{F(c) : c \in C\}$. Also let $(H)_2$ be the 2-section of H and $L(H^*)$ be the line graph of the dual of H. Then $(H)_2 = L(H^*)$.*

Proof. Let $F(c) = (A(c), B(c))$ be an h-part of the soft hypergraph H for some $c \in C$. Then its 2-section $[F(c)]_2$ has vertex set $A(c)$ which becomes the hyperedge set in the in the dual $[F^*(c)]$ of $F(c)$. So $A(c)$ will become the vertex set of $L[F^*(c)]$. That is, $[F(c)]_2$ and $L[F^*(c)]$ have the same vertex set $A(c)$. Two vertices in $[F(c)]_2$ are adjacent if and only if they belong to a common hyperedge in the h-part $F(c)$, which occurs if and only if the respective hyperedges intersect in the dual hypergraph $F^*(c)$; this happens if and only if the respective vertices in $L[F^*(c)]$ are adjacent. That is, $[F(c)]_2 = L[F^*(c)]$. Hence $\{[F(c)]_2 : c \in C\} = \{L[F^*(c)] : c \in C\}$. That is, $(H)_2 = L(H^*)$. □

Corollary 10.3. *For any soft hypergraph H , $L(H) = (H^*)_2$.*

Proof. We apply the equality $(H^*)^* = H$ in the above theorem to get this result. □

11. Conclusion

Soft hypergraph was introduced by applying the concept of soft set in hypergraph. By means of parameterization, soft hypergraph produces a series of descriptions of a complicated relation described using a hypergraph. Definitely, theory of soft hypergraphs will become an important part in hypergraph theory due to its capability to deal with the parameterization tool.

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