23114613

QP CODE: 23114613

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) SPECIAL SUPPLEMENTARY EXAMINATIONS, APRIL 2023

Fifth Semester

Bachelor of Sports Management

CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2020 Admission Only

68DAE135

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define finite and infinite sets with proper examples?
2. Define ϵ neighbourhood of a point? If x belongs to $V_\epsilon(a)$ for all $\epsilon > 0$ then prove that $x = a$?
3. Define supremum, infemum of sets? What is the supremum and infemum of the set $A = \{\frac{1}{n} : n \in N\}$?
4. Define rational numbers in terms of the decimal expansion? Find the decimal representation of $-\frac{2}{7}$?
5. Find $lim(\frac{2n}{n+2})$.
6. If X = (x_n) is a convergent sequence of real numbers such that $x_n \geq 0$ for every n, then prove that x = lim(x_n) $\geq$0.
7. Prove that $lim(\frac{2n}{n^2+1}) = 0$.

8. Use the recurrance relation of n^{th} term of a sequence that converges to $\sqrt{a}$ to find the value of $\sqrt{2}$ correct to 4 decimal places.

9. Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$ for all n. Prove that if $\lim x_n = +\infty$ then $\lim y_n = +\infty$.

10. Show that the series $\Sigma \frac{\cos n}{n^2}$ is convergent.

11. Establish the convergence or divergence of the series $\sum \frac{1}{(n+1)(n+2)}$.

12. Show that $\lim\limits_{x \to c} x^3 = c^3$ for any $c \in \mathscr{R}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that If A, B are bounded sets then $Sup\ (A+B) = Sup\ A + Sup\ B$ where $A+B = \{a+b : a \in A, b \in B\}$

14. If $I_n = [a_n, b_n], n \in N$ be a nested sequence of closed, bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $Inf\{b_n - a_n : n \in N\} = 0$, then Prove that the number η contained in $I_n\ \forall\ n$ is unique?

15. Prove that the m-tail of a sequence converges if and only if the sequence converges.

16. State and prove Bolzano-Weierstrass Theorem.

17. State and prove Cauchy Convergence Criterion.

18. Give an example of a convergent series Σa_n so that Σa_n^2 is not convergent.

19. State and prove Alternating Series Test.

20. Evaluate the one-sided limits of the function $h(x) = \frac{1}{(e^{\frac{1}{x}}+1)}$ at $x = 0$.

21. Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, $c \in \mathscr{R}$ be a cluster point of A. If $f(x) \leq g(x)$ for all $x \in A, x \neq c$,Then prove the following

(a) If $\lim\limits_{x \to c} f = \infty$, then $\lim\limits_{x \to c} g = \infty$.

(b) If $\lim\limits_{x \to c} g = -\infty$, then $\lim\limits_{x \to c} f = -\infty$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that the set of all real numbers is a complete ordered field?

23. (a) State and prove Monotone Convergence Theorem.

    (b) Let Z = (z_n) be the sequence defined as z_1 = 1 and $z_{n+1} = \sqrt{2z_n}$ forevery n. Show that lim(z_n) = 2.

24. Test the convergence and absolute convergence of the following series.

    1. $\sum_1^\infty \frac{(-1)^{n+1}}{(n+1)}$
    2. Whose nth term is $\frac{(-1)^n n^n}{(n+1)^{n+1}}$

25. (a) Let $A \subseteq \mathscr{R}$, $f : A \to \mathscr{R}$ and let $c \in \mathscr{R}$ be a cluster point of A. If $a \leq f(x) \leq b$ for all $x \in A, x \neq c$, and if $\lim_{x\to c} f$ exists,Then prove that $a \leq \lim_{x\to c} f \leq b$.

    (b) Check whether the following limits exist or not. Give explanations

    (1) $\lim_{x\to 0} cos(\frac{1}{x})$ (2) $\lim_{x\to 0} xcos(\frac{1}{x})$

(2×15=30)