QP CODE: 23106362

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) IMPROVEMENT / REAPPEARANCE EXAMINATIONS, MARCH 2023

Fourth Semester

CORE COURSE - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission Onwards

A2088DDF

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find parametric equations for the line through $P(-3, 2, -3)$ and $Q(1, -1, 4)$.
2. Write the vector equation and the simplified component equation for a plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$.
3. With proper justification, find the point(s) of discontinuity, if any, for the function $\mathbf{g}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + \lfloor t \rfloor \mathbf{k}$.Here $\lfloor\ \rfloor$ denotes the greatest integer function.
4. Find the potential function for the field $F(x, y, z) = 2xi + 3yj + 4zk$.
5. Find the parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.
6. State Stokes Theorem.
7. If $a \equiv b \pmod{n}$,prove that $gcd(a, n) = gcd(b, n)$.
8. Check whether the integer 1729 is an *absolute pseudoprime* or not.
9. Calculate $\phi(360)$ and $\phi(1001)$.
10. State Existence theorem for Laplace Transforms.
11. Prove that the inverse Laplace Transform is a linear operation.
12. Find $\mathscr{L}^{-1}\left\{\frac{1}{s^2+\frac{s}{2}}\right\}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Define the gradient vector of a function in the plane. Represent the directional derivative of a differentiable function in the plane as a dot product. Also, give the properties of the directional derivative.

14. The cylinder $f(x, y, z) = x^2 + y^2 - 2 = 0$ and the plane $g(x, y, z) = x + z - 4 = 0$ meet in an ellipse E. Find parametric equations for the line tangent to E at the point $P_0(1, 1, 3)$.

15. Evaluate the line integral of $f(x, y, z) = ye^x$ along the curve $r(t) = (4t)i - (3t)j, -1 \leq t \leq 2$.

16. Evaluate the integral $\int(xyz)dz$ along the curve $r(t) = costi + sintj - costk, 0 \leq t \leq \pi$.

17. Find the work done by $F = (4x - 2y)i + (2x - 4y)j$ in moving a particle once counter clockwise around the circle $(x-2)^2 + (y-2)^2 = 4$.

18. Prove: If p and q are distinct primes with $a^p \equiv a \pmod{q}$ and $a^q \equiv a \pmod{p}$ then $a^{pq} \equiv a \pmod{pq}$.

19. Let a and b are integers that are not divisible by the prime p, then if $a^p \equiv b^p \pmod{p}$ prove that $a^p \equiv b^p \pmod{p^2}$.

20. Solve $y'' + y = 2\,t, y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}, y'\left(\frac{\pi}{4}\right) = 2 - \sqrt{2}$ using Laplace Transform.

21. Using convolution theorem, solve $y'' + 5y' + 4y = 2\,e^{-2t},\ y(0) = 0,\ y'(0) = 0.$

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.
    1. Find and graph the **osculating circle** of the parabola $y = x^2$ at the origin.
    2. Find the **curvature** for the helix $\mathbf{r}(t) = (a\cos t)\mathbf{i} + (a\sin t)\mathbf{j} + bt\mathbf{k}, a, b \geq 0, a^2 + b^2 \neq 0.$

23. Verify any one form of Green's Theorem for the vector field $F(x, y) = (x - y)i + xj$ and the region R bounded by the unit circle $C : r(t) = (cost)i + (sint)j$, $0 \leq t \leq 2\pi$.

24.

1. State and prove Wilson's theorem.
2. Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, where p is an odd prime, has a solution if and only if $p \equiv 1 \pmod{4}$.

25.

1. State and prove Existence theorem for Laplace Transforms.
2. Find $\mathscr{L}^{-1}\left\{\frac{5s^2+3s-16}{(s-1)(s-2)(s+3)}\right\}$.

(2×15=30)