QP CODE: 23004830

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Reg No : ....

Name : ....

MSc DEGREE (CSS) EXAMINATION , JULY 2023

Second Semester

CORE - ME010202 - ADVANCED TOPOLOGY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

2588B36E

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Show that in a Hausdorff space X, a point x and a finite subset not containing x can be separated using disjoint open sets in X
2. Show that "if at all an extension of a map f: A→R exists its values on closure of A are uniquely determined by the value on A; where A is subset of a space X"
3. Define the term standard base and standard sub-base for the product topology.
4. Show that projection functions are open ?
5. What is the relation between productive property, countably productive property and finitely productive property?
6. Characterise evaluation function.
7. Characterise (a)Tychonoff space and (b) Completely regular spce.
8. Prove that the union of a locally finite family of closed sets is a closed set.
9. Suppose S: D → X is a net and F is a cofinal subset of D. If S/F : F → X converges to a point x in X, show that x is a cluster point of S.
10. Define path homotopy between two paths f and g in a space X

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Show that a connected T_4 space with atleast two points must be uncountable

12. Let A be a closed subset of a normal space X and suppose $f : A \to (-1, 1)$ is a continuous function. Show that there exists a continuous extension of f to X

13. Prove that a subset of X is a large box if and only if it is the intersection of finitely many walls.

14. Prove that a product of topological spaces is completely regular if and only if each coordinate space is completely regular

15. Obtain a condition under which the the evaluation function is one-to-one. State the definition used in the proof.

16. Prove that every countably compact metric space is second countable.

17. (a) Define Directed set
    (b) Define a net
    (c) Give one example for each

18. Let S: D→ X is a net in a topological space X and let x in X. Show that x is a cluster point of S if there exists a subnet of S which converges to x in X

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. State and prove Tychonoff theorem on normality and hence deduce that every regular second countable space is normal

20. a) Define the product of an indexed family of sets starting from the definition of cartesian product of finite sets.
    b) Define projection function on product space and give an example
    c) Describe the products $1)[0,1] \times [0,1]$ and $2)\{1,2,3\} \times R$ in the cartesian plane

21. Prove that in a metric space compactness, countable compactness and sequential compactness are equivalent.

22. a) Define the convergence of net in a space
    b) Characterise Hausdorff spaces in terms of convergence of nets in it
    c) Give an example to show that the term net cannot be replaced by sequence in question b.

(2×5=10 weightage)