23124785

QP CODE: 23124785

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, MAY 2023

Second Semester

CORE COURSE - MM2CRT01 - MATHEMATICS - ANALYTIC GEOMETRY, TRIGONOMETRY AND DIFFERENTIAL CALCULUS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 ADMISSION ONWARDS

703CBE60

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Derive the equation of chord of contact of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
2. Find the condition that the lines lx + my + n = 0 and $l_1x + m_1y + n_1 = 0$ to be conjugate with respect to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
3. Prove that the tangents at the extremities of a diameter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, are parallel to the diameter conjugate to it.
4. Define equi-conjugate diameters. Find the length of each equi-conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
5. Find the coordinates at the extremities of the latus rectum of a parabola $\frac{l}{r} = 1 + \cos\theta$.
6. Find the equation for a circle centered at the pole. Give an example.
7. Prove that $\sin 3x = 3 \sin x - 4 \sin^3 x$.
8. Prove that $\sin h\, 2x = 2 \sinh x \cosh x$.
9. Separate into real and imaginary parts $cot(\alpha + i\beta)$.

10. If $y = \frac{x^5}{Ax^2+2Bx+C}$, show that
$(Ax^2 + 2Bx + C)y_n + 2n(Ax + B)y_{n-1} + n(n-1)Ay_{n-2} = 0$ if n>5.

11. Determine $lim[\frac{1}{x-2} - \frac{1}{log(x-1)}]$ as $x \to 2$.

12. Evaluate $lim_{x\to 0}(cosx)^{\frac{1}{x^2}}$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the locus of the point of intersection of two tangents to the parabola $y^2 = 4ax$, which makes an angle α with one another.

14. Find the orthoptic locus of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

15. Show that the locus of the middle points of the chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touching the ellipse $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ is $A^2.\frac{x^2}{a^4} + B^2.\frac{y^2}{b^4} = (\frac{x^2}{a^2} + \frac{y^2}{b^2})^2$.

16. Show that the locus of the poles of normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^6}{x^2} + \frac{b^6}{y^2} = (a^2 - b^2)^2$.

17. Replace the polar equation $r = \frac{4}{2\cos\theta - \sin\theta}$ by equivalent cartesian equation, and identify its graph.

18. Sum the series $sin\alpha - \frac{sin(\alpha+2\beta)}{2!} + \frac{sin(\alpha+4\beta)}{4!} - \ldots$

19. Sum the series $1 + ccosh\alpha + c^2 cosh2\alpha + \ldots + c^{n-1}cosh(n-1)\alpha$., where c is less than unity.

20. Find the nth derivative of $e^{ax}cos(bx + c)$.

21. Find the nth derivative of $\frac{10x-21}{(2x-3)(2x+5)}$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Find th econdition that teh line lx + my + n = 0 is a tangent to

a) the parabola $y^2 = 4ax$

b) the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ c) the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

23. Show that the locus of the intersection of perpendicular tangents to a conic is a circle.

24. Factorize the expression $x^n - 1$

25. (a) If $p^2 = a^2 cos^2\theta + b^2 sin^2\theta$,prove that $p + \frac{d^2p}{d\theta^2} = \frac{a^2b^2}{p^3}$.

(b) If $y = Ax + Blogx$, show that $x^2 log(\frac{x}{e})\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$.

(2×15=30)