QP CODE: 23004012

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, JUNE 2023

Fourth Semester

Elective - ME800403 - COMBINATORICS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

1283AE94

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Find the number of common positive divisors of* 10^{40} *and* 20^{30}.
2. *There are 7 boys and 3 girls in a gathering. In how many ways can they be arranged in a row so that the girls form a single block*
3. A 4-storey house is to be painted by some 6 differenet colours such that each storey is painted in one colour.How many ways are there to paint the house
4. Explain Pigeonhole Principle
5. Stating necessary theorems Show that R(3,q)$\leq \frac{1}{2}(q^2+q)\ \forall q \geq 2$
6. Let A and B be finite sets. Show that $|\bar{A} \bigcap B| = |B| - |A \bigcap B|$
7. Find the number of integers which are divisible by 5 but not by 7 in $\{1,2,\dots,200\}$
8. State simplest form of PIE?.Write the general form of PIE using n finte numberr of sets
9. Find the exponential generating function for (a_r) where a_r is the number of r-permutations of p identical objects.
10. Define recurrence relation for a sequence(a_n) with an example.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Define pairing of a 2n element set.Find number of pairing of a 2n element set in two different ways
12. Find the non negetive integer solution to the equation $5x_1 + x_2 + x_3 + x_4 = 14$

13. Six points are there in general position in the space.The 15 line segments joining them in pairs are drawn and painted with some segments with blue and rest of them with red.Prove that there is some triangle has all its sides are of same colour

14. For all $p, q \geq 2$ and if the Ramsay numbers $R(p-1, q) and R(p, q-1)$are both even derive a sharper upper bound for $R(p, q)$

15. Find the number of integer solutions of the equation $x_1 + x_2 + x + 3 = 20$ *where* $0 \leq x_1 \leq 3, 0 \leq x_2 \leq 3, 0 \leq x_3 \leq 11$ using GPIE

16. Prove that $\sum_{k=0}^{m}(-1)^k\binom{m}{k}(m-k)^n = 0 \; if \; n < m$

17. Let $k, n \in N$ with $k \leq n$. Then Show that the number of partition of n ito k parts is equal to number of partition of n into parts the largest size of which is k

18. Solve the recurrence relation $a_n - 7a_{n-1} + 15a_{n-2} - 9a_{n-3} = 0$, given that $a_0 = 1, a_1 = 2$ and $a_2 = 3$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. A) Find the number of $r-$element subset of $\{1, 2, 3, \ldots, n\}$ that contains no concecutive numbers

B)Find the number of elements in the power set of a set containing n distinct elements

20. Let ABC be an equilateral triangle and E the set of all points contained in the 3 segments AB,BC,CA (including A,B and C). Show that for every partition of E into 2 disjoint subsets, at least one of the 2 subsets contains the vertices of a right angled triangle.

21. Derive the formula for $D(n, r, k)$ and hence show that $D(n, r, k) = \binom{r}{k}D(n-k, r-k, 0)$

22. (a) Define ordinary generating function for a sequence (a_r).

(b) Find the coefficient of x^9 *and* x^{14}, in the expansion of $(1 + x +^2 x^3 + x^4 + x^5)^4$

(c) Express the generating function for the sequence (c_r) where $c_r = \sum_{i=1}^{r} i^2$ for each $r \in N^*$ in a form not involving any series.And hence show that $\sum_{i=1}^{r} i^2 = \binom{r+1}{3} + \binom{r+2}{3}$

(2×5=10 weightage)