23004011

QP CODE: 23004011

Reg No : .....

Name : .....

M Sc DEGREE (CSS) EXAMINATION, JUNE 2023

Fourth Semester

Elective - ME800402 - ALGORITHMIC GRAPH THEORY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

A6A68D3E

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define block and end block in a graph. Give examples.
2. Determine the order, size and degree of regularity of the $n-$cube, Q_n.
3. What is tractable problem,intractable problem?
4. Define a tree. Draw all trees of order 5.
5. Define a rooted tree. Give an example.
6. Define radius and diameter of a graph G. Give example.
7. Define flow in a network N
8. Define vertex disjoint $u-v$ paths in a graph G and the term $\kappa(u,v)$, if $u,v \in V(G)$
9. Let l be a feasible vertex labeling of a weighted complete bipartite graph G. If H_l contains a perfect matching M', then prove that M' ia a maximum weight matching of G.
10. *State Kirkman's school girl problem*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. (a) Define isomorphism of two graphs. Give an example of a graph G of order 5 such that $G \cong \bar{G}$.
(b) Define equal graphs. Are two equal graphs isomorphic? Is the coverse true? Justify.

12. Explain Greedy algorithm , find its complexity

13. Prove that Kruskal's algorithm produces a minimum spanning tree in a non-trivial connected weighted graph

14. Define the distance function on a graph G. Write an algorithm to find $d(u, v)$ for a fixed vertex u and every vertex v of G.

15. Write an algorithm to determine a Maximum flow and minimum cut in a network N with underlying digraph D ,source s , sink t , capacity function c , and initial flow f , where f could be the zero flow.

16. For every graph G , show that $\kappa(G) \leq \lambda(G) \leq \delta(G)$

17. *Prove that every $r-$regular bipartite multigraph, $r \geq 1$ has a perfect matching.*

18. *Prove that Petersen graph is not 1-factorable*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Havel – Hakimi Theorem

b) Determine whether the following sequence is graphical and , if so, find a graph having s as its degree sequence.

$s : 5, 5, 4, 4, 3, 3, 3, 1, 0, 0$

20. Explain Depth First Search Algorithm using an example. Find its complexity.

21. (a) Let N be a network with underlying digraph D, source s, sink t, capacity function c, and flow f. Let D' be the digraph with the same vertex as D and arc set

$$E(D') = \{(x, y); (x, y) \in E(D) \text{ and } c(x, y) > f(x, y), \text{ or } (y, x) \in E(D) \text{ and } f(y, x) > 0\}$$

Prove that D' contains a (directed) s-t path if and only if D contains an $f-$augmenting semipath.

(b) Draw the network with underlying digraph D, source s, sink t . Vertex set $V(D) = \{s, u, v, w, x, y, t\}$. Arcs, capacity and flow of each arc, are given in the following table.

Use the Max-flow Min-Cut Algorithm to find the maximum flow and minimum cut.

Arcs	(s,u)	(s,x)	(w,s)	(u,v)	(x,v)	(x,y)	(y,w)	(y,t)	(v,t)	(w,t)
Capacity	3	4	3	2	1	3	2	1	3	5
Flow	1	4	1	1	1	3	2	1	2	1

22. *Prove that a non trivial graph G has a 1-factor if and only if for every proper subset S of $V(G)$, the number of odd components of $G - S$ does not exceed $|S|$*

(2×5=10 weightage)