23123802

QP CODE: 23123802

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR EXAMINATIONS, MAY 2023

Fourth Semester

CORE COURSE- MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2021 Admission Only

A045E911

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Write the component equation and simplified component equation for a plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$.

2. Find the arc length parameter along the helix $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$ from t_0 to t.

3. Define the **tangent plane** at a point on a smooth surface in space. Give the formula for the plane tangent to a surface $z = f(x, y)$ at $(x_0, y_0, f(x_0, y_0))$.

4. Define simple closed curve in a plane.

5. State any one form of Green's Theorem.

6. Find the divergence of the vector field $F = -yi + xj$.

7. Define a complete set of residues modulo n.

8. Use Fermat's theorem to verify that 17 divides $11^{104} + 1$.

9. Define *pseudoprime*.

10. Find $\mathscr{L}^{-1}\left\{\frac{s+3}{(s-1)(s+2)}\right\}$.

11. Find $\mathscr{L}(e^{-t}\sinh t)$.

12. Define the convolution of the functions $f(t)$ and $g(t)$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. A helicopter is to fly directly from a helipad at the origin in the direction of the point $(1,1,1)$ at a speed of 60 ft/sec. What is the position of the helicopter after 10 sec?
14. State and prove the **Chain Rule** for differentiating vector functions.
15. Evaluate the line integral of $f(x,y,z) = ye^x$ along the curve $r(t) = (4t)i - (3t)j, -1 \leq t \leq 2$.
16. Find a potential function f for the vector field $F = (ysinz)i + (xsinz)j + (xycosz)k$.
17. Explain the parameters of spherical co-ordinate system and find the parametrization of the sphere of radius a centerd at the origin $(0,0,0)$.
18. Prove: If p and q are distinct primes with $a^p \equiv a \pmod q$ and $a^q \equiv a \pmod p$ then $a^{pq} \equiv a \pmod{pq}$.
19. If$d|n$, then prove that $\phi(d)|\phi(n)$.
20. Find $\mathscr{L}^{-1}\left\{\frac{1}{s^2(s^2+\omega^2)}\right\}$.
21. Solve $y'' - \frac{1}{4}y = 0,\ y(0) = 4,\ y'(0) = 0$ using Laplace Transform.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.
    1. Define the gradient vector of a function in the plane. Find an equation for the tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at the point $(-2,1)$.
    2. Find the derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $P_0(1,1,0)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. In what direction does f change most rapidly at P_0, and what are the rates of change in these directions?

23. State Stoke's Theorem and use it to evaluate the flux of the curl of the field $F = 2zi + 3xj + 5yk$ across the surface $R(r,\theta) = (rcos\theta)i + (rsin\theta)j + (4-r^2)k$, $0 \leq r \leq 2, 0 \leq \theta \leq 2\pi$

24.

1. State and prove Wilson's theorem.
2. Prove the converse of Wilson's theorem.

25.

1. Using Laplace Transform, solve
   $y'' + 2y' + 5y = 50\,t - 150,\ y(3) = -4,\ y'(3) = 14.$
2. Solve the Volterra integral equation of the second kind
   $y(t) - \int_0^t y(\tau)\sin(t-\tau)d\tau = t.$

(2×15=30)