23127310

QP CODE: 23127310

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, OCTOBER 2023

Third Semester

Core Course - MM3CRT01 - CALCULUS

Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

AB9FB874

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Show that $y = \log x$ is everywhere conve downwards.
2. Find the radius of the curvature $y^2 = x^3$ at the point (4,8).
3. Define centre of curvature at any point p of a curve.
4. Find the envelope of the family of the semi-cubical parabola $y^2 = (x+a)^2$.
5. Evaluate f_{xy} if $f(x,y) = \sqrt{x^2+y^2}$
6. Find $\frac{dw}{dt}$ if $w = xy + z,\ x = \cos t,\ y = \sin t\ z = t$
7. State the second derivative test for local maximum values of a function $f(x,y)$ at (a,b).
8. Evaluate the volume of the solid of cross sectional area $A(x) = 5x^2$ from $x = 0$ to $x = 1$.
9. Find the length of the curve $y = x^{\frac{3}{2}}$ from $x = 0$ to $x = 4$.
10. The line segment $x = 1 - y;\ 0 \leq y \leq 1$ is revolved about the Y-axis to generate the cone. Find its lateral surface area (which excludes base area).
11. Find the average value of $f(x,y) = \sin(x+y)$ over the rectangle $0 \leq x \leq \pi;\ 0 \leq y \leq \pi$.

12. Express the spherical coordinate (ρ, ϕ, θ) in terms of rectangular coordinates (x, y, z)

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Using Maclaurin's series expand $\dfrac{e^x}{\cos x}$ around x=0 .

14. Expand $\log(x+k)$ in powers of x using Taylor's series.

15. If $u = \ln \dfrac{x^2+y^2}{x+y}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$

16. Find the absolute maximum and minimum values $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by the lines $x = 0,\ y = 0,\ y = 9 - x.$

17. The region between the curve $y = \sqrt{x}$; $0 \le x \le 4$ and the X-axis is revolved about the X-axis to generate a solid . Find its volume.

18. Find the volume of the solid generated by revolving each region in the first quadrant bounded above by the curve $y = x^2$, below by the X-axis and on the right by the line $x = 1$, about the line $x = -1.$

19. Sketch the region of integration and write an equivalent double integral of $\displaystyle\int_0^2 \int_{x^2}^{2x} (4x+3)\ dy\, dx$ with the order of integration reversed.

20. Find the average value $f(x, y, z) = xyz$ over the cubical region D bounded by the coordinate planes $x = 2, y = 2$ and $z = 2$ in the first octant.

21. Find the Jacobian $J(\rho, \phi, \theta)$ for the transformation $x = \rho \sin\phi \cos\theta, y = \rho \sin\phi \sin\theta, z = \rho \cos\phi$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Find all the asymptotesof the curve $y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x^2 + 2y - 2x - 1 = 0$

23. (a). Find the positive numbers x, y, z such that $xyz = 64$ and $x + y + z$ is minimum.
(b). The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.

24. (a). The region enclosed by the X-axis and the parabola $y = 2x - x^2$ is revolved about the vertical line $x = -1$ to generate a solid. Find the volume of the solid using shell method.

(b). Find the length of the curve $y = \frac{x^3}{12} + \frac{1}{x}$ from $x = 0$ to $x = 4$.

25.
(a). Evaluate $\int_0^1 \int_0^{1-x^2} \int_3^{(4-x^2-y)} x \, dz \, dy \, dx$

(b). Evaluate the cylindrical coordinate integral $\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} dz \, r \, dr \, d\theta$

(2×15=30)