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QP CODE: 23144636

Reg No : .....

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**M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2023****Third Semester**

Faculty of Science

**CORE - ME010303 - MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS**

M Sc MATHEMATICS, M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

59928569

Time: 3 Hours

Weightage: 30

**Part A (Short Answer Questions)***Answer any **eight** questions.**Weight **1** each.*

1. Define Exponential Fourier transform, Fourier sine transform, Fourier cosine transform, Laplace transform and Mellin transform.
2. Define convolution of  $f$  and  $g$ . Also show by an example that Lebesgue integrability of  $f$  and  $g$  alone will not give a convolution integral of  $f$  and  $g$ .
3. Show that if  $\mathbf{F}(t) = \mathbf{f}(\mathbf{c} + t\mathbf{u})$ , then  $\mathbf{F}'(t) = \mathbf{f}'(\mathbf{c} + t\mathbf{u}; \mathbf{u})$ , if either derivative exists.
4. Show that the existence of total derivative of a function  $\mathbf{f} : S \rightarrow \mathbf{R}^m$ ,  $S \subseteq \mathbf{R}^n$  at  $\mathbf{c} \in S$ , implies the existence of directional derivative  $\mathbf{f}'(\mathbf{c}; \mathbf{u}) \forall \mathbf{u} \in \mathbf{R}^n$ .
5. Let  $\mathbf{f} : S \rightarrow \mathbf{R}^m$  be differentiable at each point of an open connected subset  $S$  of  $\mathbf{R}^n$ . Show that if  $\mathbf{f}'(\mathbf{c}) = 0; \forall \mathbf{c} \in S$ , then  $\mathbf{f}$  is constant on  $S$ .
6. Let  $f(z) = e^z$ . Verify that  $J_f(z) \neq 0$  for all  $z$  in  $C$  but  $f$  is not one-one.
7. State inverse function theorem.
8. Define a Stationary point and a Saddle point.
9. Define flip of a linear operator. Give an example.
10. Define differential form of order  $k$ . Write standard presentation of  $\omega = x_1 dx_2 \wedge dx_1 \wedge dx_3 - x_2 dx_3 \wedge dx_2 \wedge dx_1$ .

(8×1=8 weightage)



### Part B (Short Essay/Problems)

Answer any **six** questions.

Weight 2 each.

11. Let  $f$  be a real valued and continuous function on  $[a, b]$ . Then for every  $\epsilon > 0$ , there is a polynomial  $p$ , such that  $|f(x) - p(x)| < \epsilon$  for every  $x$  in  $[a, b]$ .
12. Find the Fourier series for  $f(x) = \frac{x^2}{8}, 0 \leq x \leq 2\pi$ .
13. a. Define matrix of linear function.  
b. Show that if  $S, T$  are linear functions with domain of  $S$  containing range of  $T$  the matrix of composite function  $SoT$ , is the product of matrices of linear functions  $S, T$ .
14. Compute the gradient vector  $\nabla f(x, y)$  at those points  $(x, y) \in \mathbf{R}^2$  if  
a.  $f(x, y) = x^2 y^2 \log(x^2 + y^2)$  if  $(x, y) \neq (0, 0), f(0, 0) = 0$ .  
b.  $f(x, y) = xy \sin(x^2 + y^2)$
15. Define the Jacobian determinant of a function  $f$  on  $\mathbf{R}^n$ .  
If  $f = u + iv$  is a complex valued function with a derivative at a point  $z$  in  $C$ , Prove that  $J_f(z) = |f'(z)|^2$
16. (a) Define Quadratic form. When will you say that a quadratic form is positive definite.  
(b) Find the saddle point of the function  $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$
17. For every  $f \in C(I^k)$ , show that  $L(f) = L'(f)$ .
18. (a) If  $\gamma(t) = (a \cos t, b \sin t), 0 \leq t \leq 2\pi$ , then find  $\int_{\gamma} x dy$  and  $\int_{\gamma} y dx$ .  
(b) If  $\Phi(r, \theta, \phi) = (x, y, z)$  where  $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$  and  $D$  is the 3-cell defined by  $0 \leq r \leq 1, 0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi$ , then show that  $\int_D dx \wedge dy \wedge dz = \frac{4\pi}{3}$ .  
(6×2=12 weightage)

### Part C (Essay Type Questions)

Answer any **two** questions.

Weight 5 each.

19. Prove that the Fourier transform of a convolution  $f * g$  is the product of the convolutions of  $f$  and of  $g$ .
20. State and prove the matrix form of the chain rule.
21. (a) If both partial derivatives  $D_r f$  and  $D_k f$  exist in an  $n$ -ball  $B(c)$  and if both  $D_{r,k} f$  and  $D_{k,r} f$  are continuous at  $c$ . Prove that  $D_{r,k} f(c) = D_{k,r} f(c)$ .  
(b) If both partial derivatives  $D_r f$  and  $D_k f$  exist in an  $n$ -ball  $B(c; \delta)$  and if both are differentiable at  $c$ . Prove that  $D_{r,k} f(c) = D_{k,r} f(c)$ .



22. Suppose  $K$  is a compact subset of  $\mathbb{R}^n$  and  $\{V_\alpha\}$  is an open cover of  $K$ .

Show that there exist functions  $\psi_1, \psi_2, \dots, \psi_s \in C(\mathbb{R}^n)$  such that

a)  $0 \leq \psi_i \leq 1$  for  $1 \leq i \leq s$

b) Each  $\psi_i$  has its support in some  $V_\alpha$ .

c)  $\psi_1(x) + \psi_2(x) + \dots + \psi_s(x) = 1, \forall x \in K$ .

(2×5=10 weightage)