QP CODE: 23144634

23144634

Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2023

Third Semester

Faculty of Science

CORE - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

C88C9D0C

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. *Define orthogonal trajectories on the surface of a system of curves.*
2. *Eliminate the arbitrary function* f *from the equation* $f(x^2+y^2+z^2,\ z^2-2xy)=0$
3. What is the general form of the linear partial differential equation in n variables. Explain how a general solution of this equation is found.
4. Verify that the equation $z=\sqrt{(2x+a)}+\sqrt{2y+b}$ is a complete integral of the partial differential equation $z=\frac{1}{p}+\frac{1}{q}$.
5. Find a complete integral of the equation $(p+q)(z-xp-yq)=1$.
6. Verify that the partial differential equation $t=a^2r$ is satisfied by $z=f(x+ay)+g(x-ay)$.
7. Define reducible linear differential operator.
8. Show that $F(D,D')e^{ax+by}\phi(x,y)=e^{ax+by}F(D+a,D'+b)\phi(x,y)$.
9. Define a family of equipotential surfaces and corresponding potential function.
10. Show that the real and imaginary parts of an analytic function are harmonic

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. *Find the integral curves of* $\frac{dx}{y(x+y)+az}=\frac{dy}{x(x+y)-az}=\frac{dz}{z(x+y)}$

12. *Verify that the equation* $(x^2z - y^3)\,dx + 3xy^2\,dy + x^3\,dz = 0$ *is integrable and if so find its primitive.*

13. Find the complete integral of the equation $z^2 = pqxy$.

14. Find the complete integral of the equation $(p^2 + q^2)y = qz$.

15. Solve by Jacobi's method $p^2x + q^2y = z$.

16. Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$.

17. By separating the variables solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

18. Show that in cylindrical coordinates ρ, z, ϕ, the Laplace's equation has solutions of the form $R(\rho)exp(\pm mz \pm in\phi)$ where $R(\rho)$ is a solution of Bessel's equation $\frac{d^2R}{d\rho^2} + \frac{1}{\rho}\frac{dR}{d\rho} + (m^2 - \frac{n^2}{\rho^2})R = 0$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. Prove the following.

a) A necessary and sufficient condition that the Pfaffian differential equation $X \cdot dr = 0$ should be integrable is that $X \cdot curl X = 0$.

b) Given one integrating factor of the Pfaffian differential equation $X_1dx_1 + X_2dx_2 + \cdots + X_ndx_n = 0$, *we can find an infinity of them.*

20. Find the general equation of the surfaces orthogonal to the family given by $x(x^2 + y^2 + z^2) = c_1y^2$ showing that one such orthogonal set consists of the the family of spheres given by $x^2 + y^2 + z^2 = c_2z$. If a family exists, orthogonal to both the above equations, show that it must satisfy $2x(x^2 - z^2)dx + y(3x^2 + y^2 - z^2)dy + 2z(2x^2 + y^2)dz = 0$.

21. Reduce the equation to canonical form and solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$.

22. (a) State and prove the necessary condition that a family of surfaces $f(x, y, z) = c$ is a family of equipotential surfaces

(b) Show that the surfaces $(x^2 + y^2)^2 - 2a^2(x^2 - y^2) + a^4 = c$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.

(2×5=10 weightage)