QP CODE: 23144632

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2023

Third Semester

Faculty of Science

CORE - ME010301 - ADVANCED COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

161866B3

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Check whether $1 < |z| < 2$ is a symmetric region.
2. Explain what is meant by Harnack's inequality.
3. Find the poles of $\pi cot \pi z$ and the corresponding singular parts.
4. Prove that $\Pi_2^\infty(1 - \frac{1}{n^2}) = \frac{1}{2}$.
5. State Hadamard's theorem.
6. *Find a relation between* $\zeta(s)$ *and* $\Gamma(1-s)$ *for* $\sigma > 1$.
7. *Prove that* $\xi(s) = \frac{1}{2}s(1-s)\pi^{-(\frac{s}{2})}\Gamma(\frac{s}{2})\zeta(s)$ *is entire and satisfies* $\xi(s) = \xi(1-s)$.
8. *Prove that a family of functions is normal then its closure with respect to distance function is compact.*
9. What is meant by the modular group?
10. Prove that $\wp'(z) = -\frac{\sigma(2z)}{\sigma(z)^4}$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any ***six*** *questions.*

Weight ***2*** *each.*

11. (a) Suppose that $u(z)$ is harmonic for $|z| < R$ and continuous for $|z| \leq R$. Prove that $u(a) = \frac{1}{2\pi} \int\limits_{|z|=R} \frac{(R^2-|a|^2)}{|z-a|^2} u(z)d\theta$, for all $|a| < R$.

    (b) If $|a| < R$, then evaluate $\int\limits_{|z|=R} \frac{(R^2-|a|^2)}{|z-a|^2} d\theta$.

12. *Prove that if* V_1 *and* V_2 *are subharmonic then* $V = Max\{V_1, V_2\}$ *is subharmonic.*

13. State and prove Weirstrass' theorem for convergence of analytic functions.

14. Write the general form of a Laurent series for $f(z)$ which is analytic in the annulus $R_1 < |z-a| < R_2$. Derive the Laurent series of $f(z) = \frac{1}{z^2(1-z)}$ in $0 < |z| < 1$.

15. *State and prove a connection between* $\zeta(s)$ *and the ascending sequence of primes.*

16. *Does zeta function have any zero? Justify.*

17. Let f be a topological mapping of a region Ω onto a region Ω'. If $\{z_n\}$ or $z(t)$ tends to the boundary of Ω, then prove that $\{f(z_n)\}$ or $f(z(t))$ tends to the boundary of Ω'.

18. Prove that $\sigma(z+\omega_1) = -\sigma(z)e^{\eta_1(z+\frac{\omega_1}{2})}$ and $\sigma(z+\omega_2) = -\sigma(z)e^{\eta_2(z+\frac{\omega_2}{2})}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any ***two*** *questions.*

Weight ***5*** *each.*

19. (a) If u_1 and u_2 are hormonic in a region Ω, then prove that $\int_\gamma u_1.^* du_2 - u_2.^* du_1 = 0$.

    (b) Deduce that $\int_\gamma (u_1 \frac{\partial u_2}{\partial n} - u_2 \frac{\partial u_1}{\partial n})|dz| = 0$.

20. Define the Gamma function. Prove that $\Gamma(z) = \frac{e^{-\gamma z}}{z} \Pi_{n=1}^{\infty} (1+\frac{z}{n})^{-1} e^{\frac{z}{n}}$. Also show that $\Gamma(z)\Gamma(z+\frac{1}{2}) = e^{az+b}\Gamma(2z)$ where a and b are constants.

21. *Prove the necessary and sufficient condition for a family* $\mathcal{F}$ *of continuous functions with values in a metric space S to be normal in a region* Ω *of the complex plane.*

22. (a) Define the Riemann mapping. Prove that Riemann mapping establishes a conformal mapping from the unit disk onto any simply connected region other than the plane itself.

    (b) Prove that the Riemann mapping is unique.

(2×5=10 weightage)