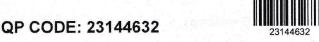


Weightage: 30



Reg No :

M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2023

Third Semester

Faculty of Science

CORE - ME010301 - ADVANCED COMPLEX ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)
2019 ADMISSION ONWARDS
161866B3

Time: 3 Hours

Part A (Short Answer Questions)

Answer any **eight** questions.

Weight **1** each.

1. Check whether 1 < |z| < 2 is a symmetric region.

- 2. Explain what is meant by Harnack's inequality.
- 3. Find the poles of $\pi cot\pi z$ and the corresponding singular parts.
- 4. Prove that $\Pi_2^{\infty}(1-\frac{1}{n^2})=\frac{1}{2}$.
- 5. State Hadamard's theorem.
- 6. Find a relation between $\zeta(s)$ and $\Gamma(1-s)$ for $\sigma > 1$.
- 7. Prove that $\xi(s) = \frac{1}{2}s(1-s)\pi^{-(\frac{s}{2})}\Gamma(\frac{s}{2})\zeta(s)$ is entire and satisfies $\xi(s) = \xi(1-s)$.
- 8. Prove that a family of functions is normal then its closure with respect to distance function is compact.
- 9. What is meant by the modular group?
- 10. Prove that $\wp'(z) = -\frac{\sigma(2z)}{\sigma(z)^4}$.

(8×1=8 weightage)



Part B (Short Essay/Problems)

Answer any six questions.

Weight 2 each.

- 11. (a) Suppose that u(z) is harmonic for |z| < R and continuous for $|z| \le R$. Prove that $u(a) = \frac{1}{2\pi} \int\limits_{|z|=R} \frac{(R^2-|a|^2)}{|z-a|^2} u(z) d\theta,$ for all |a| < R.
 - (b) If |a| < R, then evaluate $\int\limits_{|z|=R} rac{(R^2 |a|^2)}{|z-a|^2} d heta.$
- 12. Prove that if V_1 and V_2 are subharmonic then $V = Max\{V_1, V_2\}$ is subharmonic.
- 13. State and prove Weirstrass' theorem for convergence of analytic functions.
- 14. Write the general form of a Laurent series for f(z) which is analytic in the annulus $R_1 < |z-a| < R_2$. Derive the Laurent series of $f(z) = \frac{1}{z^2(1-z)}$ in 0 < |z| < 1.
- 15. State and prove a connection between $\zeta(s)$ and the ascending sequence of primes.
- 16. Does zeta function have any zero? Justify.
- 17. Let f be a topological mapping of a region Ω onto a region Ω' . If $\{z_n\}$ or z(t) tends to the boundary of Ω , then prove that $\{f(z_n)\}$ or f(z(t)) tends to the boundary of Ω' .
- 18. Prove that $\sigma(z+\omega_1)=-\sigma(z)e^{\eta_1(z+\frac{\omega_1}{2})}$ and $\sigma(z+\omega_2)=-\sigma(z)e^{\eta_2(z+\frac{\omega_2}{2})}$

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any two questions.

Weight 5 each.

- 19. (a) If u_1 and u_2 are hormonic in a region Ω , then prove that $\int_{\gamma} u_1 \cdot du_2 u_2 \cdot du_1 = 0$. (b) Deduce that $\int_{\gamma} (u_1 \frac{\partial u_2}{\partial n} u_2 \frac{\partial u_1}{\partial n}) |dz| = 0$.
- 20. Define the Gamma function. Prove that $\Gamma(z)=\frac{e^{-\gamma z}}{z}\Pi_{n=1}^{\infty}(1+\frac{z}{n})^{-1}e^{\frac{z}{n}}$. Also show that $\Gamma(z)\Gamma(z+\frac{1}{2})=e^{az+b}\Gamma(2z)$ where a and b are constants.
- 21. Prove the necessary and sufficient condition for a family \mathcal{F} of continuous functions with values in a metric space S to be normal in a region Ω of the complex plane.
- 22. (a) Define the Riemann mapping. Prove that Riemann mapping establishes a conformal mapping from the unit disk onto any simply connected region other than the plane itself.
 - (b) Prove that the Riemann mapping is unique.

(2×5=10 weightage)