23135626

QP CODE: 23135626

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, OCTOBER 2023

Fifth Semester

CORE COURSE - MM5CRT03 - ABSTRACT ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

C91B8DE2

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. State whether the set $\mathbb{Z}^+$ under multiplication is a group. Justify.
2. Define left identity element in a group and left inverse of an element in a group.
3. Define a cyclic group.
4. Define a **permutation of a set**. Compute $\sigma\tau$ where σ and τ are permutations given by $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$.
5. Define the **orbits** of a permutation of a set.
6. Let H be a subgroup of a group G. Define the index $(G : H)$ of H in G. Give a formula to compute $(G : H)$ when G is finite.
7. Define the **Cartesian product of sets** $S_1, S_2, \cdots, S_n$.. Write the number of elements in the Cartesian product of the sets $\{0, 1\}$ and $\{0, 1, 2\}$.
8. Check whether $f : (GL_n(\mathbb{R}), .) \rightarrow (\mathbb{R}^*, .)$ defined by $f(A) = det(A)$ is a group homomorphism or not.
9. If $\phi : G \rightarrow G'$ is a group homomorphism then show that $\phi(e) = e'$ where e and e' are identity elements of G and G' respectively.
10. Compute the product in the given ring a) (2,3) (3,5) in $Z_5 \times Z_9$ b) (-3,5) (2,-4) in $Z_4 \times Z_{11}$
11. Find all units in Z_{14}.

12. Give an example to show that a factor ring of an integral domain may be a field.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Determine whether $*$ defined on $\mathbb{Z}^+$ by $a * b = a^b$ is a) commutative b) associative.

14. Check whether $\langle \mathbb{Q}, + \rangle$ and $\langle \mathbb{Z}, + \rangle$ under the usual addition are isomorphic.

15. State and prove Division Algorithm for $\mathbb{Z}$.

16. Let G and G' be groups and let $\phi : G \to G'$ be a one-to-one function such that $\phi(xy) = \phi(x)\phi(y)$ for all $x, y \in G$. Then show that $\phi[G]$ is a subgroup of G' and ϕ provides an isomorphism of G with $\phi[G]$.

17. If $n \geq 2$, then prove that the collection of all even permutations of $\{1, 2, 3, \cdots, n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group S_n.

18. Let $\phi : G \to G'$ is a group homomorphism, if N is a normal subgroup of G' show that $\phi^-1(N)$ is a normal subgroup of **G**.

19. Define maximal normal subgroup of a group.Prove that **M** is a maximal normal subgroup of a group **G** if and only if the factor group G/M is simple.

20. a) Mark each of the following true or false.

    i) Every field is an integral domain

    ii) The characteristic of nZ is n

    b) Prove that Z_p is a field if p is a prime.

21. Show that if $R, R' and R''$ are rings and if $\phi : R \to R'$ and $\tau : R' \to R''$ are homomorphism, then the composite function $\tau\phi : R \to R''$ is a homomorphism.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Find all subgroups of $\mathbb{Z}_{36}$ and draw the subgroup diagram.

23.

1. Let H be a subgroup of a group G. Let the relation $\sim_R$ be defined on G by $a \sim_R b$ if and only if $ab^{-1} \in H$. Then show that $\sim_R$ is an equivalence relation on G. What is the cell in the corresponding partition of G containing $a \in G$?

2. Let H be a subgroup of a group G. Then define the left and right cosets of H containing $a \in G$.
3. Let H be the subgroup $< \mu_1 >= \{\rho_0, \mu_1\}$ of S_3. Find the partitions of S_3 into left cosets of H.

24. State and prove fundamental homomorphism theorem.

25. a) Prove that the divisors of 0 in Z_n are those nonzero elements that are not relatively prime to n .

b) Find the divisors of Z_{16}

c) Prove that Z_p , where p is prime has no divisors of 0.

(2×15=30)