23135620

**QP CODE: 23135620**

**Reg No** : .....................

**Name** : .....................

# B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, OCTOBER 2023

## Fifth Semester

## CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

000B74C6

Time: 3 Hours Max. Marks : 80

### Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that the set of all integers $Z$ is denumerable.
2. Determine the set $A = \{x \in R : |2x + 3| < 7\}$?
3. If $t > 0$ prove that there exist an $n_t \in N$ such that $0 < \dfrac{1}{n_t} < t$
4. Justify the validity of the following statement with proper reasoning "A positive real number is rational then its decimal expansion is periodic".
5. Show that $lim(\frac{1}{n} - \frac{1}{n+1}) = 0$.
6. Find $lim((2 + \frac{1}{n})^2)$.
7. Use the recurrance relation of $n^{th}$ term of a sequence that converges to $\sqrt{a}$ to find the value of $\sqrt{2}$ correct to 4 decimal places.
8. Give an example of an unbounded sequence that has a convergent subsequence. Explain.
9. Prove that a monotone sequence of real numbers is properly divergent if and only if it is bounded.
10. Using Comparison test, discuss the convergence of $\Sigma \frac{1}{n(n+1)}$.

11. State Abel's test for the convergence of series.

12. Show that $\lim_{x \to 0}(x + sgn(x))$ do not exist.

(10×2=20)

**Part B**

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove that $a < b \iff a^2 < b^2 \iff \sqrt{a} < \sqrt{b}, a, b \geq 0$.

14. State and prove characterisation of intervals theorem.

15. Using definition of limits, prove that $lim(\frac{3n+2}{n+1}) = 3$.

16. Let X = ($x_n$) and Y = ($y_n$) be sequences of real numbers that converges to x and y respectively and c ϵ **R**. Prove that the sequences cX converges to cx.

17. Prove that every contractive sequence is Cauchy and hence is convergent.

18. State and prove the comparison test for the convergence of series. Using this test, show that $\sum_1^\infty \frac{1}{n^2+n}$ is convergent.

19. If $\Sigma a_n$ is a convergent series of real numbers then is it necessary that $\Sigma \frac{\sqrt{a_n}}{n}$ is convergent?

20. Evaluate the following one-sided limits
(a) $\lim_{x \to 1^+} \frac{x}{x-1} (x \neq 1)$.
(b) $\lim_{x \to 0^+} \frac{(x+2)}{\sqrt{x}} (x > 0)$

21. Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, $c \in \mathscr{R}$ be a cluster point of $A$. If $f(x) \leq g(x)$ for all $x \in A, x \neq c$,Then prove the following
(a) If $\lim_{x \to c} f = \infty$, then $\lim_{x \to c} g = \infty$.
(b) If $\lim_{x \to c} g = -\infty$, then $\lim_{x \to c} f = -\infty$.

(6×5=30)

**Part C**

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that there exist a real number $x$ such that $x^2 = 2$ ?

23. (a) State and prove Monotone Convergence Theorem.

(b) Let Y = $(y_n)$ be the sequence defined as $y_1 = 1$ and $y_{n+1} = \frac{2y_n+3}{4}, n \geq 1$. Prove that $\lim Y = \frac{3}{2}$.

24.

1. State and prove Rearrangement Theorem.
2. If $\Sigma a_n$ is convergent, then prove that any series obtained from it by grouping terms is also convergent to the same value.

25. (a) Let $A \subseteq \mathscr{R}$, $f : A \rightarrow \mathscr{R}$ and let $c \in \mathscr{R}$ be a cluster point of $A$. If $a \leq f(x) \leq b$ for all $x \in A, x \neq c$, and if $\lim\limits_{x \to c} f$ exists,Then prove that $a \leq \lim\limits_{x \to c} f \leq b$.

(b) Check whether the following limits exist or not. Give explanations

(1) $\lim\limits_{x \to 0} sinx$ (2) $\lim\limits_{x \to 0}(\frac{cosx-1}{x})$

(2×15=30)