23126985

QP CODE: 23126985

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, OCTOBER 2023

Third Semester

COMPLEMENTARY COURSE - MM3CMT01 - MATHEMATICS - VECTOR CALCULUS, ANALYTIC GEOMETRY AND ABSTRACT ALGEBRA

Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance

2017 Admission Onwards

DFE66A84

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the tangent to the curve $r(t) = 2sint\ i + (t^2 - cost)\ j + e^t k$ at $t_0 = 0$.
2. Define arc length parametrisation of an arc based on a fixed point on the arc.
3. Find the gradient of $f(x, y, z) = x^2 + y^2 + z^2$at $(1, 1, 1)$.
4. Find a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.
5. Find the curl of the vector field $\mathbf{F} = 2zi + 3xj + 5yk$ at the point $(1, 0, 1)$.
6. Define the divergence of a vector field in space.
7. Express the equation of the curve $x^2 + y^2 = 4$ in polar co-ordinates.
8. Find the equation of the parabola with vertex $(5, -3)$, axis parallel to the y-axis and passes through the point $(10, 2)$.
9. Find the eccentricity of the hyperbola $9x^2 - 16y^2 = 144$.
10. How many group you can define on the set G if $G = \{a, b\}$?

11. Define a cyclic group.

12. What is the order of the symmetric group S_n?

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the derivative of $f(x, y) = \frac{x-y}{xy+2}$ at $(1, -1)$ in the direction of the vector $u = 12i + 5j$.

14. Find the equation of tangent to curve $x^2 - y = 1$ at the point $(\sqrt{2}, 1)$.

15. Evaluate $\int_C y\,dx - x\,dy$ along the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$ in the counter clockwise direction.

16. Find the circulation and flux of the vector field $\mathbf{F} = xi + yj$ around and across the closed semicircular path consists of the semicircular arch $\mathbf{r}_1(t) = (a\cos t)\,i + (a\sin t)\,j,\ 0 \le t \le \pi$ followed by the line segment $\mathbf{r}_2(t) = ti,\ -a \le t \le a$.

17. Show that $\mathbf{F} = (\cos y + y\cos x)\,i + (\sin x - x\sin y)\,j$ is a conservative vector field. Also find the potential function for the fieid.

18. Find the vertices, focii, length of the semimajor axis and the length of the semiminor axis of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

19. Find the standard equation of the hyperbola with focus $(4, 0)$ and directrix $x = 2$.

20. Prove that if $< G, * >$ is an abelian group then for all $a, b \in G$ and for all integeers $(a * b)^n = a^n * b^n$.

21. Define Z_4 and Klien 4-group. Find their subgroups and draw the subgroup diagrams.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) Find the unit tangent, principal normal and curvature of the curve $r(t) = e^t cost\ i + e^t sint\ j + 2k$.

(b) Find the directions in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly and decreases most rapidly at the point $(1, 1)$.

23. Verify Gauss Divergence Theorem for $\mathbf{F} = xi + yj + zk$ where S is the surface of the cube bounded by the planes $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1$.

24. (a) Find the equation of the ellipse when $x^2 + 4y^2 = 16$ is shifted 3 units to the left and 5 units up. Also find the center,vertices, foci and directrix of the new ellipse. Sketch the new ellipse with all these details.
(b) Find the polar equation of the parabola $x^2 = -8y$.

25. Show that the set of all permutations on a three element set forms a non abelian group under permutation multiplication and also draw its subgroup diagram.

(2×15=30)