M Sc DEGREE (CSS) EXAMINATION, NOVEMBER 2023

Third Semester

Faculty of Science

CORE - ME010305 - OPTIMIZATION TECHNIQUE

M Sc MATHEMATICS, M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

0181AB58

Time: 3 Hours   Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define degenerate basic feasible solution of an LPP.

2. Write a short note on Applications of Duality.

3. *If an optimal solution of $Min f(X)$ subject to $X \in S_F$ exist and T_F is nonempty. Prove that optimal solution of $Min f(X)$ subject to $X \in T_F$ and $Min f(X)$ subject to $X \in [T_F]$ exist and optimal solution of $Min f(X)$ subject to $X \in S_F$ is a lower bound for $Min f(X)$ subject to $X \in T_F$ and $Min f(X)$ subject to $X \in [T_F]$.*

4. *What is a Pruned and fathomed solution of an ILPP?*

5. Define the following with suitable example.
   (i) Directed graph   (ii) Circuit   (iii) Tree

6. Define spanning tree with example.

7. What you mean by critical path method.

8. Define the terms (i) stationary point   (ii) global optimum   (iii) monotonic increasing sequence.

9. Derive Taylor's series.

10. Write short note about perturbation vector.

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any six questions.

Weight 2 each.

11. Show that all the basic solutions of the following LP are infeasible.
Maximize $z = x_1 + x_2$, Subject to $x_1 + 2x_2 \leq 6, 2x_1 + x_2 \geq 16, x_1 \geq 0, x_2 \geq 0.$

12. Write the dual of the following LP problem and verify that the dual of the dual is primal.
Minimize $6x_1 + 3x_2 - 2x_3$ subject to
$3x_1 + 4x_2 + x_3 \geq 5, 6x_1 - 3x_2 + x_3 \geq 2$ and $x_1, x_2, x_3 \geq 0.$

13. *Solve graphically: Min $f(X) = 2x_1 + 3x_2$ subject to $8x_1 - 4x_2 \geq 7, 3x_1 + x_2 \leq 5, x_1 \geq 0, x_2 \geq 0.$*

14. *Solve the Either Or problem:*
Maximise $2x_1 + 5x_2$ subject to $0 \leq x_1 \leq 8, 0 \leq x_2 \leq 8,$ and $4 - x_1 \geq 0$ or $4 - x_2 \geq 0.$

15. What you mean by goal programming.
A factory can manufacture two products A and B. The profit on a unit of A is Rs. 80 and of B is Rs. 40. The maximum demand of A is 6 units per week and B is 8 units per week. This manufacturer has set a goal of achieving a profit of Rs. 640 per week. Formulate the problem as goal programming and solve it.

16. State and prove maximum flow minimum cut theorem.

17. Minimize $(x_1 - 2)^2 + (x_2 - 1)^2$ subject $x_1 - 2x_2 + 1 = 0$

18. Write all Kuhn –Tucker conditions of NLP
Minimize $f(x) = x_1^2 - x_1 x_2 + 3x_2^2 - 4x_2 + 4$
subject to $g(x) : 1 - x_1 - x_2 \geq 0$ and $h(x) : 2x_1^2 + 3x_2^2 = 13.$

(6×2=12 weightage)

Part C (Essay Type Questions)

Answer any two questions.

Weight 5 each.

19. Solve the following LPP using simplex method
Minimize $f(X) = x_1 - 3x_2 + 2x_3$
Subject to
$3x_1 - x_2 + 3x_3 \leq 7, -2x_1 + 4x_2 \leq 12, -4x_1 + 3x_2 + 8x_3 \leq 10; x_1, x_2, x_3 \geq 0$

20. *Solve the ILPP Min$f(x) = 4x_1 + 5x_2$ subject to*
$3x_1 + x_2 \geq 2, x_1 + 4x_2 \geq 5, 3x_1 + 2x_2 \geq 7, x_1, x_2$ *are positive integers.*

21. Find the minimum path from v_0 to v_8..

Arc	(0,1)	(0,2)	(0,3)	(1,2)	(1,4)	(1,5)	(2,3)	(2,5)	(3,5)	(3,6)
Length	2	6	8	3	10	8	1	1	2	4
Arc	(4,5)	(4,7)	(5,4)	(5,7)	(6,5)	(6,7)	(6,8)	(7,4)	(7,6)	(7,8)
Length	1	3	1	5	4	6	7	2	1	10

22. Maximize the function $f(x) = -3x^2 + 21.6x + 1.0$ with a minimum resolution of $\epsilon = 0.5$ over 6 functional evaluations. The optimal value of $f(x)$ is assumed to lie in the range $25 \geq x \geq 0$.

(2×5=10 weightage)