QP CODE: 23145324

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, DECEMBER 2023

First Semester

CORE - ME010104 - REAL ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

223E7A6B

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

Answer any ***eight*** *questions.*

Weight ***1*** *each.*

1. Define bounded variation with an example.
2. Let f be of bounded variation on $[a, b]$ and let V be defined as $V(x) = V_f(a, x)$, if $a < x \leq b$ and $V(0) = 0$. Then prove that every point of continuity of V is a point of continuity of f
3. *Give an example of a function, $f \notin \mathscr{R}$ on $[a, b]$ for $a < b$.*
4. *If $f_1(x) \leq f_2(x)$ on $[a, b]$ then prove that* $\int_a^b f_1\, d\alpha \leq \int_a^b f_2\, d\alpha$.
5. Define the unit step function I. Is it continuous?.
6. Differentiate between pointwise convergence and uniform convergence of a sequence of functions.
7. Is every Cauchy sequence convergent? If no, when will it be convergent?
8. Under what conditions, a sequence $\{f_n\}$ of continuous functions defined on a compact set K, is convergent uniformly to a continuous function f ?
9. Define piontwise boundedness and uniform boundedness of a sequence of functions.
10. If $0 < t < 2\pi$, then prove that $e^{it} \neq 1$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

Answer any ***six*** *questions.*

Weight ***2*** *each.*

11. Let f be of bounded variation on $[a, b]$. Let V be defined on $[a, b]$ as follows: $V(x) = V_f(a, x)$ if $a < x \leq b$, $V(a) = 0$. Then prove that

(i). V is an increasing function on $[a, b]$.
(ii). $V - f$ is an increasing function on $[a, b]$.

12. Explain the terms graph, curve and path. Prove by an example that different paths can trace out the same curve.

13. *If P^* is a refinement of P, then establish a relation between $L(P, f, \alpha)$ and $L(P^*, f, \alpha)$.*

14. *State and prove the fundamental theorem of calculus.*

15. When do we say that a series of functions is convergent? Also give an example to show that a convergent series of continuous functions may have a discontinuous sum.

16. Obtain a series from $\phi(x) = |x|, (-1 \leq x \leq 1)$ and $\phi(x+2) = \phi(x)$ for all real x, which converges uniformly on R^1.

17. If f is continuous on $[0, 1]$ and if $\int_0^1 f(x)x^n \, dx = 0, \; n = 0, 1, 2, \ldots$, prove that $f(x) = 0$ on $[0, 1]$.

18. If the two series $\sum a_n x^n$ and $\sum b_n x^n$ converges in $S = (-R, R)$, $E = \{x \epsilon S : \sum a_n x^n = \sum b_n x^n\}$ and E has a limit point in S then prove that the given series is identical.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) State and prove additive property of arc length function $\Lambda_f(x, y)$ for a rectifiable curve *f*.
(ii) Define $s(x) = \Lambda_f(a, x)$ for $x \in [a, b]$ and let $s(a) = 0$ for a rectifiable path f defined on $[a, b]$. Then prove that the function f is increasing and continuous on $[a, b]$.
(iii) Let $f : [a, b] \to R^n$ and $g : [c, d] \to R^n$ be two paths in R^n, each of which is one to one on its domain. Then prove that **f** and **g** are equivalent if and only if they have the same graph.

20. *Suppose f is bounded on [a, b], f has only finitely many points of discontinuity on [a, b] and* $\boldsymbol{\alpha}$ *is continuous at every point at which f is discontinuous then, prove that* $f \in \mathcal{R}(\alpha)$.

21. Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathscr{R}(\alpha)$ on $[a, b]$, for $n = 1, 2, 3, \ldots$ and suppose $f_n \to f$ uniformly on $[a, b]$. Then prove that $f \in \mathscr{R}(\alpha)$ on $[a, b]$ and $\int_a^b f d\alpha = \lim_{n \to \infty} \int_a^b f_n d\alpha$.
Also show that if the series $f(x) = \sum_{n=1}^{\infty} f_n(x), (a \leq x \leq b)$ converges uniformly on $[a, b]$, then $\int_a^b f \, d\alpha = \sum_{n=1}^{\infty} \int_a^b f_n \, d\alpha$.

22. If K is compact, if $f_n \in \mathscr{C}(K)$ for $n = 1, 2, 3, \ldots$, and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, prove that
(i) $\{f_n\}$ is uniformly bounded on K
(ii) $\{f_n\}$ contains a uniformly convergent subsequence.

(2×5=10 weightage)