QP CODE: 23145326

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, DECEMBER 2023

First Semester

CORE - ME010105 - GRAPH THEORY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

03F40F6B

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define (a) complete bipartite graph (b) selfcomplementary graph (c) clique of a graph (d) isomorphism between graphs
2. If G is a simple graph and $\delta \geq \frac{n-1}{2}$ then show that G is connected.
3. Show that no vertex v of a simple graph can be a cut vertex of both G and G^C.
4. Define and give example for the following
   (a) nonseparable graph
   (b) block of a graph
   (c) End block of a graph
5. a. Define centre of a graph and centroid of a tree.
   b. Give an example of (i) a tree with one central vertex that is also a centroidal vertex,
   (ii) a tree with 2 centroidal vertices , one of which is also a central vertex.
6. Define a Hamiltonian graph and traceable graph. Give an example to show that a traceable graph need not be hamiltonian.
7. Describe the construction of the closure of a graph G.
8. Define proper vertex coloring and chromatic number.
9. Define a planar graph
10. Define a circulant of order n

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Show that set Aut(G) of all automorphisms of a simple graph G is a group with respect to the composition of mappings as the group operation.

12. Give an example to show that $G_1[G_2]$ need not be isomorphic to $G_2[G_1]$

13. Let set $(v_1, v_2, \ldots\ldots\ldots v_n)$, $n \geq 2$ be given and let $(d_1, d_2, \ldots\ldots\ldots d_n)$ be a sequence of positive integers such that $\sum_{i=1}^{n} d_i = 2(n-1)$ Then prove that the number of trees with $(v_1, v_2, \ldots\ldots\ldots v_n)$ as the vertex set in which v_i has degree d_i, $1 \leq i \leq n$ is $\frac{(n-2)!}{(d_1-1)!\ldots\ldots(d_n-1)!}$

14. Write Prim's algorithm for determining a minimum weight spanning tree in a connected weighted graph.

15. Draw the graph associated with Konigsberg Bridge Problem. Is the graph Eulerian. Justify the claim.

16. If G is k-critical, then prove that $\delta(G) \geq k-1$.

17. State and prove Euler formula for a connected planar graph G and prove that the number of faces is invariant under any plane embedding of G.

18. Define dual of a plane graph. Draw dual of Herschel graph.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19.
   a. Show that every tournament contains a directed Hamiltonian path.
   b. Show that every tournament of order n has at most one vertex v with $d^+(v)=n-1$.
   c. Show that every tournament T is diconnected or can be made into one by the reorientation of just one arc of T.

20.
   a. State and prove Whitney's theorem.
   b. Prove for any loopless connected graph G ,$\kappa(G) \leq \lambda(G) \leq \delta(G)$.

21. a. For any graph G with n vertices and independence number α, prove that $n/\alpha \leq \chi \leq n - \alpha + 1$.
    b. For any simple graph G, prove that $2\sqrt{n} \leq \chi + \chi^c \leq n+1$ and $n \leq \chi\chi^c \leq ((n+1)/2)^2$

22. *Prove that every planar graph is 5 – vertex colorable.*

(2×5=10 weightage)