QP CODE: 23145322

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, DECEMBER 2023

First Semester

CORE - ME010103 - BASIC TOPOLOGY

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 ADMISSION ONWARDS

D98FE0CB

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. What you mean by metrisable topology, Give an example with justification?
2. Define a base of a topological space? Justify with example
3. Define subspace of a topological space .How will you write an open set in a space using its subbase elements?Why?
4. *Let A be a subset of a Topological Space* $(X, \mathfrak{I})$. *Then Show that* $\bar{A}$ *is a closed set and is the smallest closed subset of X containing A.*
5. Define strong topology.
6. *Define quotient map and quotient topology.*
7. Define a compact subset of a topological space X and give an example
8. Define connected space. If X is a connected space, prove that X cannot be written as the disjoint union of two nonempty closed subsets .
9. Define a path,simple path,closed path in a topological space and a path connected space
10. *Show that every* T_1 *space is* T_0 *but not conversely*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that the collection of all open sets in a metric space form a topology on the same set
12. Show that there exist a unique smallest topology on a non empty set X containing any given family of subsets of X

13. Prove that a subset of a topological space is open if and only if it is a neighborhood of each of its points.

14. Let $f : X \to Y$ be a function, where X, Y are topological spaces. Then prove that the following statements are equivalent (i) f is a homeomorphism, (ii) f is a continuous bijection and f is open.(iii) $f$is a bijection and $f^{-1}$is open. (iv) There exist a function $g : Y \to X$such that f, g are continuous and $g \circ f = id_X$ and $f \circ g = id_Y$

15. Let X , Y be topological spaces,x∈X. and f: X→Y a function . Suppose X is first countable at x and f is continuous at x and { x_n } is a sequence in X .Prove that the sequence { f (x_n)} converges to f (x) in Y.

16. Let X_1, X_2 be connected topological spaces. Prove that $X_1 \times X_2$ is connected.

17. *Let Y be a Hausdorff space and X be any space.Let f and g be any two maps from X to Y. Show that the set {* $x \epsilon X : f(x) = g(x)$ *} is closed in X*

18. *Show that regularity is a hereditary property*

(6×2=12 weightage)

Part C (Essay Type Questions)

- *Answer any **two** questions.*

*Weight **5** each.*

19. Show that metrisability is a hereditary property

20. Let $f : X \to Y$ be a function, where $\mathfrak{J}, \mathcal{U}$ be topologies on X. Y respectively and $x_0 \in X$. Prove that the following statements are equivalent.(i) f is continuous at x_0, (ii) the inverse image of every neighborhood of $f(x_0)$ in Y is a neighborhood of x_0 in X, (iii) for every subset A ⊆ X. $x_0 \in \bar{A}$ implies $f(x_0) \in \overline{f(A)}$

21. *(a) Define Lebesgue number of a cover of a topological space X.*
(b) State and prove Lebesgue covering Lemma.

22. Show that locally connectedness is a divisible property

(2×5=10 weightage)