23146054

QP CODE: 23146054

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, DECEMBER 2023

First Semester

Core Course - MM1CRT01 - FOUNDATION OF MATHEMATICS

(Common to B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science, B.Sc Computer Applications Model III Triple Main)

2017 Admission Onwards

88514B45

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Give an example for Universal quantifier.
2. Define Modus tollens for propositional logic.
3. Define Universal generalization.
4. Describe what is the Cartesian product of $A_1, A_2, \ldots, A_n$.
5. Can we define the sum and product of any two functions. Define the sum and product of two functions wherever possible.
6. Given $f : A \to B$. Illustrate f^{-1} using a figure
7. Draw the diagraph that represent the relation $\{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}\ on\ \{1,2,3\}$
8. Show that the "divides" relation on the set of all positive integers is not an equivalence relation.
9. Let R be an equivalence relation on a set A.Then prove that if $[\,a\,] = [\,b\,]$ then $[a] \cap [b] \neq \phi$.
10. Frame a quartic equation with rational coefficients one of whose roots is $\sqrt{5}+\sqrt{2}$.

11. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $4x^4 - 4x^3 - 25x^2 + x + 6 = 0$,find the values of $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

12. Find the limits to the values of c such that $x^3 - 3x + c = 0$ may have all its roots real.

(10×2=20)

Part B

Answer any ***six*** *questions.*

Each question carries ***5*** *marks.*

13. Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are equivalent.

14. Show that $\neg\forall x[P(x) \rightarrow Q(x)] \equiv \exists x[P(x) \wedge \neg Q(x)]$.

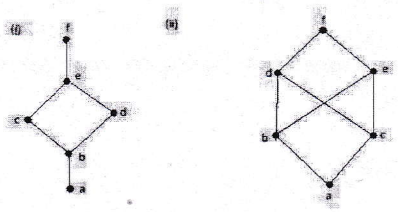
15. Show that if $'n'$ is an integer and $n^3 + 5$is odd, then $'n'$ is even by using the method of contradiction.

16. Using De Morgan's law deduce that $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$

17. Define and plot the ceiling function.

18. Suppose that the relations R and S on a set A are represented by the matrices $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.What are the matrices representing $R \cup S$ and $R \cap S$.

19. Determine whether the posets with these Hasse Diagrams are lattices.



20. Solve the equation $4x^5 + x^3 + x^2 - 3x + 1 = 0$, given that it has rational roots?

21. Solve the equation $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$?

(6×5=30)

Part C

Answer any ***two*** *questions.*

Each question carries ***15*** *marks.*

22. (a) Construct the truth table for the following compound propositions:

$(i)(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

$(ii)(p \oplus q) \rightarrow (p \wedge q)$.

(b) Use truth table to establish which of the following statements are tautologies, which are contradictions and which are contingencies.

$(i)(p \rightarrow q) \leftrightarrow (\neg p \vee q)$

$(ii)(p \wedge \neg q) \wedge (\neg p \vee q)$

$(iii)[(p \rightarrow q) \wedge \neg p] \rightarrow \neg q$

23. a) Define different set operations. Illustrate using Venn diagrams.

b) Let R_3 be the relation defined on the set of all strings S by sR_3t either when $s = t$ or both s and t are bit strings of length 3 or more that begin with the same three bits.What are the sets in the partition of the set of all bit strings arising from R_3 on the set of all bit strings?

24. (A) Let A be the set of students in a college and B be the set of books in college library.Let R_1 and R_2 be relations consisting of all ordered pairs (a,b) , where a is required to read the book b in a course and where student a has read the book b , respectively.Describe the ordered pairs in each of the following relations:

$(a)R_1 \cup R_2$ (b) $R_1 \cap R_2$ $(c)R_1 - R_2$ (d) $R_2 - R_1$ $(e)R_1 \oplus R_2$.

(B) Prove that the relation R on a set A is transitive if and only if

$R^n \subseteq R$ for $n = 1, 2, 3...$

25. a) If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, obtain the equation whose roots are $\alpha + \frac{1}{\beta\gamma}, \beta + \frac{1}{\gamma\alpha}, \gamma + \frac{1}{\alpha\beta}$.

b) Find the equation whose roots are those of $x^4 - 2x^3 + 3x - 5 = 0$ each diminished by2.

(2×15=30)