23145758

QP CODE: 23145758

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, DECEMBER 2023

First Semester

Complementary Course - MM1CMT01 - MATHEMATICS - PARTIAL DIFFERENTIATION, MATRICES, TRIGONOMETRY AND NUMERICAL METHODS

(Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III , B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance)

2017 Admission Onwards

20CEF97F

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the function value of $f(x, y, z) = \frac{x-y}{y^2+z^2}$ at $(1, \frac{1}{2}, \frac{-1}{4})$.
2. Find an equation for the level curve of the function $f(x, y) = \sqrt{x^2 - 1}$ that passes through the point $(1, 0)$.
3. If $f(x, y) = x\, e^{y^2/2}$, evaluate $\frac{\partial^5 f}{\partial x^2 y^3}$.
4. Find the rank of the matrix $\begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$
5. Define characteristic vector of a square matrix.
6. Find the characteristic values of the matrix $\begin{pmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$.
7. Prove that $\tanh(2\theta) = \frac{2\tanh\theta}{1+\tanh^2\theta}$.
8. If x is real, show that $\sinh^{-1} x = \log(x + \sqrt{x^2 + 1})$.

9. What is the real part of $\sinh(\alpha - i\beta)$?

10. Write the binomial expansion of $(1+x)^n$, when n is a positive integer and when n is a rational number.

11. Give the first and second approximations to the root of an equation $f(x) = 0$ using the bisection method.

12. Find the first approximation to a root of the equation $x^3 - 2x^2 + 3x - 5 = 0$ by the method of false position, given that the root lies between 1 and 2.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the partial derivative of the function $h(p, \phi, \theta) = p \sin\phi \cos\theta$ with respect to each variable.

14. Find all the second-order partial derivatives of $w = x \sin(x^2 y)$.

15. Find the values of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $(1,1,1)$ if $z^3 - xy + yz + y^3 - 2 = 0$.

16. Obtain the column equivalent canonical matrix A to the following matrix and hence find its rank $A = \begin{bmatrix} 3 & 1 & 2 & 5 \\ -1 & 4 & 1 & -1 \\ 1 & 9 & 4 & 3 \end{bmatrix}$

17. Verify the Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 0 & 1 & 2 \\ 3 & -3 & 2 \\ 1 & 1 & -1 \end{pmatrix}$.

18. Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of descending powers of $\cos\theta$.

19. Sum to infinity the series $1 + \frac{\cos\theta}{1!} + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots$

20. Use the Newton - Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$.

21. Use the generalized Newton's method to find a double root of the equation $f(x) = x^3 - x^2 - x + 1 = 0$ near 1.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Solve the following system of equation

$x + y - z + t = 0, x - y + 2z - t = 0, 3x + y + t = 0$

23. Solve the system of equations

$5x + 3y + 3z = 48, 2x + 6y - 3y = 18, 8x - 3y + 2z = 21$

24. (a) Expand $\sin^3\theta \cos^5\theta$ in a series of sines of multiples of θ.

(b) Sum to infinity the series $c\sin\alpha + \frac{c^2}{2}\sin 2\alpha + \frac{c^3}{3}\sin 3\alpha + \ldots$

25.

1. State the theorem, which gives a sufficient condition for convergence of the iteration process in the iteration method for finding the roots of a given equation.
2. Using the iteration method, find a real root of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0, 1]$ with an accuracy of 10^{-4}.

(2×15=30)