QP CODE: 22102720

22102720

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR EXAMINATIONS, AUGUST 2022

Fourth Semester

Complementary Course - MM4CMT01 - MATHEMATICS - FOURIER SERIES, LAPLACE TRANSFORM AND COMPLEX ANALYSIS

(Common for B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance)

2020 Admission Only

75A46DCD

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that if $f(x)$ is even, then $f^n(x)$ is even for all $n \geq 2, n \in N$(Natural Numbers)
2. What is the general expression for the Legendre polynomial $P_n(x)$ of degree n?
3. Define Laplace transform of a function.
4. If $\mathscr{L}(f) = \frac{1}{s(s^2+a^2)}$ find $f(t)$
5. Can we solve all types of differential equations using Laplace Transforms? If not, specify the types?
6. Find the real and imaginary parts of $\frac{z_1}{z_2}$, where $z_1 = 2 + 3i$ and $z_2 = 1 + i$.
7. Find the value of i^{101}.
8. Define a harmonic function.
9. Evaluate $(-3)^{3-i}$.
10. Define simple closed contour.
11. Can we conclude that the integral of $f(z) = \frac{1}{z^2+4}$ to be zero using Cauchy's integral theorem taken over the contour |z - 2|= 2. Give reason.
12. State Morera's Theorem.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Expand $f(x) = x, 0 < x < 2$ in half range (a.) Sine Series (b.) Cosine Series

14. Apply power series method to solve $y'' + y = 0$

15. Evaluate $\mathscr{L}^{-1}\{\frac{4}{(s+1)(s+2)}\}$

16. Evaluate $\mathscr{L}(t \sin at)$ by differentiation method

17. Show that $|z_1 + z_2| \leq 3$, where $z_1 = \frac{1}{2} - \frac{\sqrt{3}}{2}i$ and $z_2 = \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$.

18. Find the derivative of $\frac{(z-i)}{(z+i)}$ at the point $z = i$.

19. Prove that $|sinz|^2 = sin^2x + sinh^2y$.

20. Evaluate $\oint_C \frac{z^2-1}{z^2+1}dz$ using Cauchy's integral formula, C is the circle |z| = 1/2.

21. Evaluate $\oint_C \frac{z^4-3z^2+6}{(z+i)^3}dz$, C is the circle |z| = 3/2.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a.) Obtain the Fourier Series expansion of $f(x) = \begin{cases} \text{-a}, & -\pi < x < 0 \\ \text{a}, & 0 < x < \pi \end{cases}$ and using this deduce the expansion of $\frac{\pi}{4}$?
(b.) Obtain the Fourier Series expansion of $f(x) = x^2, 0 \leq x \leq 2\pi$ where $f(x + 2\pi) = f(x)$

23. Evaluate (a) $\mathscr{L}(t^n e^{at})$ (b) $\mathscr{L}^{-1}\{\log(1 - \frac{a^2}{s^2})\}$

24. Find and plot all the 8^{th} roots of unity.

25. Integrate f(z) = Re z along
a) the line segment from z=0 to z = 1+i.
b) the real axis from 0 to 1 and then along a straight line parallel to the imaginary axis from 1 to 1+i.

(2×15=30)