22103101

**QP CODE: 22103101**

**Reg No** : .....................

**Name** : .....................

# B.Sc DEGREE (CBCS) REGULAR / IMPROVEMENT / REAPPEARANCE EXAMINATIONS, OCTOBER 2022

## Second Semester

## Complementary Course - MM2CMT01 - MATHEMATICS - INTEGRAL CALCULUS AND DIFFERENTIAL EQUATIONS

(Common for B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry,B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology Model I, B.Sc Geology and Water Management Model III, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance)

2017 ADMISSION ONWARDS

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Time: 3 Hours　　　　Max. Marks : 80

**Part A**

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. The solid lies between the planes perpendicular to the x-axis at x = 0 and x = 4. The cross-sections perpendicular to the axis on the interval 0 $\leq$ x $\leq$ 4 are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$ . Find the volume of the solid.
2. Find the volume of the solid generated by revolving the region bounded by the curve y = $x^2$ and the lines y = 0, x = 2 about the x-axis.
3. The region in the first quadrant enclosed by the parabola y = $x^2$, the y-axis and th eline y = 1 is revolved about the line $x = \frac{3}{2}$ to generate a solid. Find the volume of the solid.
4. Evaluate the double integral $\iint_R y^2x \, dA$ over the rectangle $R = \{(x, y) : -3 \leq x \leq 2, 0 \leq y \leq 1\}$.
5. Use a double integral to find the volume of the solid that is bounded above by the plane $z = 4 - x - y$ and below by the rectangle $R = [0, 1] \times [0, 2]$.

6. Use a double integral to find the area of the region enclosed between the parabola $y = \frac{1}{2}x^2$ and the line $y = 2x$.

7. Verify that the function $x^2 + y^2 = c$ is a solution of the differential equation $y\frac{dy}{dx} + x = 0$.

8. Examine whether the differential equation $(x^2 - y^2)dx + (x^2 - 2xy)dy = 0$is exact or not.

9. Solve the differential equation $xdy - ydx = 0$.

10. Write the general form of the integral curves of the set of equations $\dfrac{dx}{P} = \dfrac{dy}{Q} = \dfrac{dz}{R}$.

11. Define partial differential equations with examples

12. Find the general integral of the linear partial differential equation $xp + yq = z$

(10×2=20)

**Part B**

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Find the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$, from x = 0 to x = 2.

14. Find the area of the surface generated by revolving the curve $x = \frac{y^4}{4} + \frac{1}{8y^2}$, 0 ≤y ≤ 2, about the x-axis.

15. Find the average value of $f(x, y) = \sin(x + y)$ over the rectangle $0 \leq x \leq \pi,\ 0 \leq y \leq \pi$.

16. Use a triple integral to find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

17. Find values of A and B so that the function $y(x) = Asinx + Bcosx + 1$ satisfy the initial conditions $y(\pi) = 0, y'(\pi) = 0$.

18. Solve $ylogydx + (x - logy)dy = 0$.

19. Solve $x\frac{dy}{dx} + y = y^2logx$

20. Find the integral curves of the equations $\dfrac{dx}{z^2 - 2yz - y^2} = \dfrac{dy}{xy + zx} = \dfrac{dz}{xy - zx}$.

21. Form the partial differential equation by eliminating the arbitrary constants from $z = ax + by + \sqrt{a^2 + b^2}$

(6×5=30)

**Part C**

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. (a) The region bounded by the curve $y = \sqrt{4x - x^2}$, the x-axis and the line x = 2 is revolved about x-axis to generate a solid. Find the volume of the solid.
(b) Find the volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{x}$, the x-axis and the line x = 4 about (i) x-axis (ii) y-axis.

23. (i) Evaluate $\int_0^1 \int_{\frac{y}{2}}^1 e^{x^2} \, dxdy.$
(ii) Change the order of integration and hence evaluate the double integral $\int_0^1 \int_x^{2-x} \frac{x}{y} \, dydx.$

24. a) Solve $x^2(y+1)dx + y^2(x-1)dy = 0.$
b)Solve $4xdy - ydx = x^2 dy.$

25. Show that the condition that the surfaces $F(x, y, z) = 0,\ G(x, y, z) = 0$ should touch is that the eliminant of $x, y$ and $z$ from these equations and the equations $\frac{F_x}{G_x} = \frac{F_y}{G_y} = \frac{F_z}{G_z}$ should hold. Hence find the condition that the plane $lx + my + nz + p = 0$ should touch the central conicoid $ax^2 + by^2 + cz^2 = 1.$

(2×15=30)