



22100530

QP CODE: 22100530

Reg No :

Name :

**B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS,
APRIL 2022**

Third Semester

**COMPLEMENTARY COURSE - MM3CMT01 - MATHEMATICS - VECTOR CALCULUS,
ANALYTIC GEOMETRY AND ABSTRACT ALGEBRA**

Common to B.Sc Chemistry Model I, B.Sc Chemistry Model II Industrial Chemistry, B.Sc Chemistry Model III Petrochemicals, B.Sc Electronics and Computer Maintenance Model III, B.Sc Food Science & Quality Control Model III, B.Sc Geology and Water Management Model III, B.Sc Geology Model I, B.Sc Physics Model I, B.Sc Physics Model II Applied Electronics, B.Sc Physics Model II Computer Applications, B.Sc Physics Model III Electronic Equipment Maintenance

2017 Admission Onwards

F768D534

Time: 3 Hours

Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find the velocity and acceleration of the particle whose position is given by $r(t) = (t - \sin t)i + (1 - \cos t)j$ at $t = \pi$.
2. Define unit tangent vector of a curve.
3. Write the formula for finding directional derivative in terms of gradient
4. Define the line integral of a continuous vector function F over a smooth oriented curve C in space.
5. Find a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.
6. State Gauss Divergence Theorem.
7. Find the equation of the hyperbola with foci $(0, \pm 4)$ vertices $(0, \pm 5)$ in standard form.
8. Find the cartesian equation of the line $r \cos(\theta - \frac{\pi}{4}) = \sqrt{2}$.





9. Find the eccentricity of the hyperbola $8x^2 - 2y^2 = 16$.
10. Give an example of a group which is commutative.
11. Define a cyclic Group.
12. Define permutation on a non empty set.

(10×2=20)

Part B

Answer any **six** questions.

Each question carries **5** marks.

13. Find the derivative of $f(x, y) = \frac{x-y}{xy+2}$ at $(1, -1)$ in the direction of the vector $u = 12i + 5j$.
14. Find the gradient of $f(x, y, z) = \ln(2x + 3y + 6z)$ at $(1, 1, 0)$ and find the derivative of the function f at this point in the direction of $n = 2i + 3j + 6j$.
15. Find the work done by the force field $\mathbf{F} = (x^2 + y) i + (y^2 + x) j + ze^z k$ along the helix $\mathbf{r}(t) = (\cos t) i + (\sin t) j + \frac{t}{2\pi} k$, $0 \leq t \leq 2\pi$ from $(1, 0, 0)$ to $(1, 0, 1)$.
16. Apply Green's Theorem to evaluate $\oint_C x \cos y dx - y \sin x dy$ where C is the square with vertices $(0, 0)$, $(\pi, 0)$, (π, π) , $(0, \pi)$ oriented counterclockwise.
17. Find the surface area cut from the bottom of the paraboloid $z = x^2 + y^2$ by the plane $z = 4$.
18. Describe the graph of $r^2 = 4r \sin \theta$.
19. Find the vertices, foci, length of the semimajor axis and the length of the semiminor axis of the ellipse $\frac{x^2}{9} + \frac{y^2}{36} = 1$.
20. Define the group Z_6 under addition modulo 8, and draw the subgroup diagram for it.
21. If p and q are distinct prime numbers, find the number of generators of Z_{pq} .

(6×5=30)

Part C

Answer any **two** questions.

Each question carries **15** marks.





22. (a) Find the unit tangent, principal normal and curvature of the curve
 $r(t) = (\cos t + t \sin t) i + (\sin t - t \cos t) j, t > 0$.
(b) Find the directions in which $h(x, y, z) = \cos(xy) + e^{yz} + \ln(zx)$ increases most rapidly and decreases most rapidly at the point $(1, 0, \frac{1}{2})$.
23. Verify Green's Theorem in the plane for $\mathbf{F} = (3x^2 - 8y^2) i + (4y - 6xy) j$ defined in the region bounded by the lines $x = 0, y = 0, x + y = 1$.
24. (a) Find the equation of the parabola $y^2 = 8x$ when shifted right 1 unit and down 2 units. Also find the vertex, focus and directrix of the new parabola. Sketch the new parabola with all these details.
(b) Find the polar equation of the circle $(x - 6)^2 + y^2 = 36$.
25. (a) Show that a group with no proper nontrivial subgroup is cyclic.
(b) Let H and K be two subgroup of a group G . Show that $H \cap K$ is a subgroup of G .

(2×15=30)

