

QP CODE: 19102120



Reg No : .....

Name : .....

**B.Sc. DEGREE (CBCS) EXAMINATION, OCTOBER 2019**

**Third Semester**

**CORE COURSE - MM3CRT01 - CALCULUS**

(Common to B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission Onwards

24232AFB

Maximum Marks: 80

Time: 3 Hours

**Part A**

*Answer any ten questions.*

*Each question carries 2 marks.*

1. Expand  $a^x$  by Maclaurin's series.
2. Write the co-ordinates of the centre of curvature of a curve  $y = f(x)$  at a point  $P(x, y)$
3. What is an oblique asymptote.
4. Find the envelope of family of straight lines  $x \cos \alpha + y \sin \alpha = a$ ,  $\alpha$  being the parameter.
5. Find  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  if  $f(x, y) = y^x$
6. Find  $\frac{dw}{dt}$  if  $w = x^2 + y^2$ ,  $x = \cos t$ ,  $y = \sin t$
7. Explain the absolute maximum of a continuous function at a point  $(a, b)$  defined on a bounded region  $R$ .
8. The solid lies between planes perpendicular to the X-axis at  $x=0$  and  $x=4$ . The cross-sections perpendicular to X-axis are squares whose diagonals run from the parabola  $y = -\sqrt{x}$  to the parabola  $y = \sqrt{x}$ . Find the area of cross section  $A(x)$ .
9. Find the volume of solid of revolution generated by rotating the region between the Y-axis and graph of the function  $y = x$ ;  $0 \leq y \leq 1$  about Y-axis.
10. Write the equations for finding surface area of revolution about (i) the X-axis (ii) the Y-axis.
11. Evaluate  $\iint_R (10 + x^2 + 3y^2) dA$  where  $R: 0 \leq x \leq 1; 0 \leq y \leq 2$
12. Evaluate  $\int_0^2 \int_0^2 \int_0^2 dz dy dx$ .

(10×2=20)

**Part B**

*Answer any six questions.*

*Each question carries 5 marks*

13. Obtain Taylor series expansion in powers of  $h$  for  $f(x) = \cos(x + h)$

14. Find the radius of curvature of  $\frac{x^3}{6} + \frac{y^3}{18} = 2$  at (3, 6)
15. Verify that  $w_{xy} = w_{yx}$  where  $w = x^2 \tan(xy)$ .
16. Find all local extreme values and saddle point, if any, of the function  $f(x, y) = x^3 - y^3 - 2xy + 6$ .
17. Find the volume of the solid generated by revolving the region bounded by the curves and lines  $y = x^2$ ,  $y = 2 - x$ ,  $x = 0$  for  $x \geq 0$  about the Y-axis using shell method.
18. Find the length of the curve  $y = \int_0^x \tan t \, dt$ ,  $0 \leq x \leq \pi/6$
19. Sketch the region of integration and calculate  $\iint_R \frac{\sin x}{x} \, dA$  where R is the triangle in the XY-plane bounded by the X-axis and the line  $y = x$  and
20. Sketch the region bounded by the lines  $x = 0$ ,  $y = 2x$  and  $y = x^2$  express the region's area as double integral and evaluate the integral.
21. Evaluate the cylindrical coordinate integral  $\int_0^{2\pi} \int_0^3 \int_{r^2/3}^{\sqrt{18-r^2}} dz \, r \, dr \, d\theta$

(6×5=30)

### Part C

Answer any two questions.

Each question carries 15 marks.

22. Find the ranges of values  $x$  in which the curve  $y = 3x^3 - 40x^2 + 3x - 20$  are concave upwards or downwards. Also find their point of inflection, equation of the inflectional tangents to the curve and show that they lie on a straight line.
23. (a). If  $\sin u = \frac{x+y}{\sqrt{x} + \sqrt{y}}$ , prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan u$ .
- (b). Find the maximum and minimum values that the function  $f(x, y) = 3x + 4y$  takes on the circle  $x^2 + y^2 = 1$
24. (a). Find the volume of the solid that results when the region enclosed by  $y = \sqrt{x}$ ,  $y = 0$  and  $x = 9$  revolved about the line  $x = 9$ .
- (b) Find the length of the curve  $x = \frac{1}{3}(y^2 + 2)^{3/2}$  from  $y = 0$  to  $y = 1$ .
- (c). Find the area of the surface generated by revolving the curve  $y = \sqrt{x} - \frac{1}{3}x^{3/2}$ ;  $1 \leq x \leq 3$ , about the X-axis.
25. (a). Evaluate  $\iint_R e^{x^2+y^2} \, dA$  where R is the semi circular region bounded by the X-axis and the curve  $y = \sqrt{1-x^2}$ .
- (b). Find the Jacobian  $\frac{\partial(x, y, z)}{\partial(u, v, w)}$  for the transformation  $u = x + y + z$ ,  $v = x + y - z$ ,  $w = x - y + z$ .

(2×15=30)