QP CODE: 20100916

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) EXAMINATION, MARCH 2020

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission onwards

1A6C9E8C

Time: 3 Hours | Marks: 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find parametric equations for the line through $(-2, 0, 4)$ parallel to $\mathbf{v} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.
2. Define limit of a vector-valued function $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$.
3. Define the length of a smooth curve $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k},\ a \le t \le b$. Also write the formula for length in terms of the length of a velocity vector $d\mathbf{r}/dt$.
4. Define flow integral along a curve.
5. Define divergence of a vector field.
6. State Divergence Theorem.
7. True or False: " If $ca \equiv cb \pmod{n}$for some integer c then $a \equiv b \pmod{n}$".Give justifications.
8. Verify that $5^{38} \equiv 4 \pmod{11}$ using Fermat's theorem.
9. If $gcd(a, 35) = 1$,show that $a^{12} \equiv 1 \pmod{35}$
10. Prove that the inverse Laplace Transform is a linear operation.
11. Find $\mathscr{L}(e^{-t}\sinh t)$.
12. Find $\mathscr{L}^{-1}\left\{\frac{s-6}{(s-1)^2+4}\right\}$.

(10×2=20)

Turn Over

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Define the **directional derivative** of a function in the plane. Using the definition, find the derivative of $f(x,y) = x^2 + xy$ at $P_0(1,2)$ in the direction of the unit vector $\mathbf{u} = (1/\sqrt{2})\mathbf{i} + (1/\sqrt{2})\mathbf{j}$.

14. The cylinder $f(x,y,z) = x^2 + y^2 - 2 = 0$ and the plane $g(x,y,z) = x + z - 4 = 0$ meet in an ellipse E. Find parametric equations for the line tangent to E at the point $P_0(1,1,3)$.

15. Find the line integral of $f(x,y,z) = x + y + z$ over the straight line segment from $(1,2,3)$ to $(0,-1,1)$.

16. Evaluate the line integral $\int (2x cosy)dx - (x^2 siny)dy$ along the parabola $y = (x-1)^2$ from $(1,0)$ to $(0,1)$.

17. Using spherical co-ordinate system, find the surface area of a sphere of radius a.

18. Let n be a composite square-free integer, say, $n = p_1 p_2 \dots p_r$,where the p_i are distinct primes.

    If $p_i - 1 | (n-1)$ for $i = 1,2,\dots,r$, then prove that n is an absolute pseudoprime.

19. Let a, b, c are integers then prove that $gcd(a,bc) = 1$ if and only if $gcd(a,b) = 1$ and $gcd(a,c) = 1$.

20. If $\mathscr{L}(f(t)) = F(s)$ and c is any positive constant, show that $\mathscr{L}(f(ct)) = \frac{1}{c}F(\frac{s}{c})$.

21. Using convolution theorem, solve $y'' + y = \sin t,\ y(0) = 0,\ y'(0) = 0$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.

1. Define the **principal unit normal** vector for a smooth plane curve. Find the principal unit normal vector for the circular motion $\mathbf{r}(t) = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j}$.
2. Find and graph the **osculating circle** of the parabola $y = x^2$ at the origin.

23. Verify Stokes Theorem for the hemisphere $S : x^2 + y^2 + z^2 = 9$, $z \geq 0$ its boundary circle $C : x^2 + y^2 = 9, z = 0$and the field $F = yi - xj$.

24. 1. State and prove Wilson's theorem.

2. If p is a prime, prove that for any integer a, $p|a^p + (p-1)!a$.

25. Using Laplace transform, solve $y(t) - \int_0^t y(\tau)\, d\tau = 1$.

(2×15=30)