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QP CODE: 19102565

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Reg No :

Name :

BSc DEGREE (CBCS) EXAMINATION, OCTOBER 2019**Fifth Semester****Core Course - MM5CRT01 - MATHEMATICAL ANALYSIS**

B.Sc Computer Applications Model III Triple Main ,B.Sc Mathematics Model I,B.Sc Mathematics Model II

Computer Science

2017 Admission Onwards

819A8253

Maximum Marks: 80

Time: 3 Hours

Part A*Answer any **ten** questions.**Each question carries 2 marks.*

1. Let $S = \{1, 2\}$ and $T = \{a, b, c\}$ then determine all the injections from S to T ?
2. Define the notion of inequality between two real numbers in terms of the positive set P in the ordering property of real numbers?
3. Define bounded and unbounded sets with proper examples?
4. Define rational numbers in terms of the decimal expansion? Find the decimal representation of $-\frac{2}{7}$?
5. If $0 < b < 1$, prove that $\lim(b^n) = 0$.
6. Define bounded sequence. Give an example.
7. Find the limit of $(3n^{1/2n})$.
8. Define Cauchy sequence. Give an example.
9. Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$ for all n . Prove that if $\lim x_n = +\infty$ then $\lim y_n = +\infty$.
10. State the Limit Comparison Test for the series.
11. Is the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ is absolutely convergent or not? Why?
12. Show that $\lim_{x \rightarrow c} x = c$ for any $c \in \mathcal{R}$.

(10×2=20)



Part B

Answer any six questions.

Each question carries 5 marks.

13. Show that for all $a, b \in \mathbb{R}$
- (a.) $\max\{a, b\} = \frac{1}{2}(a + b + |a - b|)$
- (b.) $\min\{a, b\} = \frac{1}{2}(a + b - |a - b|)$
14. Prove that If A, B are bounded sets then $\sup(A + B) = \sup A + \sup B$ where $A + B = \{a + b : a \in A, b \in B\}$
15. Prove that $\lim(n^{1/n}) = 1$.
16. Prove that $\lim(\frac{\sin n}{n}) = 0$.
17. State and prove Monotone Convergence Theorem.
18. State and prove the root test for the absolute convergence of a series in \mathbb{R} .
19. State and prove Abel's Lemma.
20. Prove that $\lim_{x \rightarrow 0} \cos(\frac{1}{x})$ does not exist but that $\lim_{x \rightarrow 0} x \cos(\frac{1}{x}) = 0$.
21. Check whether the one-sided limits of the function $g(x) = e^{\frac{1}{x}}$ at $x = 0$ exist or not.

(6×5=

Part C

Answer any two questions.

Each question carries 15 marks.

22. (a.) State and Prove Nested interval property?
- (b.) Prove that the set of real numbers is not countable?
23. (a) State Monotone Convergence Theorem.
- (b) Prove that for any real number $a > 0$, there exists a sequence (s_n) of real numbers that converges to \sqrt{a} .
- (c) Use the above sequence to evaluate the value of $\sqrt{5}$ correct to 5 decimal places.
24. Test the convergence and absolute convergence of the following series.
- $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)}$
 - Whose n th term is $\frac{(-1)^n n^n}{(n+1)^{n+1}}$
25. (a) Let $A \subseteq \mathbb{R}$, $f, g : A \rightarrow \mathbb{R}$, and let $c \in \mathbb{R}$ be a cluster point of A , Suppose that $f(x) \leq g(x)$ for all $x \in A$, $x \neq c$, Then prove the following



- If $\lim_{x \rightarrow c} f = \infty$, then $\lim_{x \rightarrow c} g = \infty$.
- If $\lim_{x \rightarrow c} g = -\infty$, then $\lim_{x \rightarrow c} f = -\infty$.

(b) Give an example of a function that has a right-hand limit but not a left-hand limit at a point.

(c) Evaluate the limit or show that it do not exist " $\lim_{x \rightarrow 1} \frac{x}{x-1}$ where $x \neq 1$.

(2×15=30)

