22100166

QP CODE: 22100166

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, JANUARY 2022

Fifth Semester

CORE COURSE - MM5CRT01 - MATHEMATICAL ANALYSIS

Common for B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science & B.Sc Computer Applications Model III Triple Main

2017 Admission Onwards

6583E7F5

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Prove that if A is a set with m elements and B is a set with n elemnents, Then $A \cup B$ has $m+n$ elements?
2. Prove that $a.a = a \implies a = 0\ or\ a = 1$?
3. If $t > 0$ prove that there exist an $n_t \in N$ such that $0 < \frac{1}{n_t} < t$
4. Define periodic and terminating decimals? Is every rational terminating? Justify?
5. Prove that the sequence (0,2,0,2,0,2,...) does not converge.
6. If $X = (x_n)$ is a sequence of real numbers, (a_n) is a sequence of positive real numbers such that lim(a_n) = 0 and if for some positive constant C > 0 and m ϵ N we have $|x_n - x| < Ca_n$ forevery n $\geq$m, then prove that lim(x_n) = x.
7. Let X = (2, 4, 6,..., 2n,...) and Y = $\left(1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\right)$. Find X + Y and X.Y.
8. Write a short note on Euler number.
9. Let (x_n) and (y_n) be two sequences of real numbers and suppose that $x_n \leq y_n$ for all n. Prove that if lim y_n = -∞ then lim x_n = -∞.
10. State the root test for the absolute convergence of a series in R.
11. State Abel's test for the convergence of series.

12. Let A be a set consisting of all rational numbers in $[0, 1]$. Then what are the cluster points of A.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Prove the following, For all $a, b \in R$
(a.) $|a + b| \leq |a| + |b|$
(b.) $||a| - |b|| \leq |a - b|$
(c.) $|a - b| \leq |a| + |b|$

14. If $I_n = [a_n, b_n], n \in N$ be a nested sequence of closed, bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $Inf\{b_n - a_n : n \in N\} = 0$, then Prove that the number η contained in $I_n \ \forall \ n$ is unique?

15. Let X = (x_n) be a sequence of non-negative real numbers. Prove that the sequence $(\sqrt{x_n})$ of positive square roots converges to $\sqrt{x}$.

16. State and prove Monotone Subsequence Theorem.

17. State and prove Cauchy Convergence Criterion.

18. Prove that if $\sum x_n$ is convergent, then any series obtained from it by grouping the terms is also convergent.

19. Discuss the convergence of the series whose nth term is $\frac{n^n}{(n+1)^{n+1}}$

20. Let $f : A \to \mathscr{R}$ and let $c \in \mathscr{R}$ be a cluster point of A. If $\lim\limits_{x \to c} f$ exists, Prove that $\lim\limits_{x \to c} |f| = |\lim\limits_{x \to c} f|$.

21. Give an example of a function that has a left-hand limit but not a right-hand limit at a point.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Prove that there exist a real number x such that $x^2 = 2$?

23. (a) State and prove Monotone Convergence Theorem.
(b) Let Z = (z_n) be the sequence defined as z_1 = 1 and z_{n+1} = $\sqrt{2z_n}$ forevery n. Show that lim(z_n) = 2.

24. (a) State and prove Comparison Test for the convergence of series. (b) Discuss the convergence of

- $\sum_1^\infty \frac{1}{n^2+n}$
- $\sum_1^\infty \frac{1}{n!}$

25. (a) Let $A \subseteq \mathscr{R}$, $f, g : A \to \mathscr{R}$, and let $c \in \mathscr{R}$ be a cluster point of A, Suppose that $f(x) \leq g(x)$ for all $x \in A$, $x \neq c$, Then prove the following

- If $\lim\limits_{x \to c} f = \infty$, then $\lim\limits_{x \to c} g = \infty$.
- If $\lim\limits_{x \to c} g = -\infty$,then $\lim\limits_{x \to c} f = -\infty$.

(b) Give an example of a function that has a left-hand limit but not a right-hand limit at a point.

(c) Evaluate the limit or show that it do not exist " $\lim\limits_{x \to 1} \frac{x}{x-1}$ where $x \neq 1$.

(2×15=30)