22100168

QP CODE: 22100168

Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, JANUARY 2022

Fifth Semester

CORE COURSE - MM5CRT03 - ABSTRACT ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

74B7AFB8

Time: 3 Hours | Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. State whether True or False:
   a) "Each element of a group appear once and only once in each row and column of the group table".
   b) "There is only one group of three elements , upto isomorphism".
2. Find the gcd of 32 and 24.
3. Find all orders of subgroups of the group $\mathbb{Z}_{17}$.
4. Find the number of elements in the set $\{\sigma \in S_4 | \sigma(3) = 3\}$.
5. Define the left regular representation of a group G.
6. Find all orbits of the permutation $\sigma : \mathbb{Z} \to \mathbb{Z}$ where $\sigma(n) = n - 3$.
7. Define the direct product of the groups $G_1, G_2, \cdots, G_n$.
8. Let G be a group. If $\phi : G \to G$ defined by $\phi(g) = g^2$ is a group homomorphism, show that G is Abelian.
9. Show that every group of order 101 is simple.
10. Prove that $\phi_a : F \to R$ by $\phi_a(f) = f(a)$ for $f \in F$, F is the ring of all function mapping R into R is a ring homomorphsim.
11. Solve the equation 3x = 2 in the field Z and in the field Z $_{23}$.

12. Define a) Kernel of a ring homomorphism

b) Ideal of a ring

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Determine whether $*$ defined on $\mathbb{Q}$ by $a * b = ab/2$ is a) commutative b) associative.

14. Show that the subset S of $M_n\ (R\ )$ consisting of all invertible $n \times n$ matrices under matrix multiplication is a group.
Also check whether it is an abelian group.

15. Let G be the multiplicative group of all invertible $n \times n$ matrices with entries in $\mathbb{C}$ and let T be the subset of G
consisting of those matrices with determinant 1. Then prove that T is a subgroup of G.

16. Prove that for $n \geq 2$, the number of even permutations in S_n is the same as the number of odd permutations. Define the alternating group A_n on n letters.

17. Prove that every group of prime order is cyclic.

18. Let G be a group and order of G is a prime number. Show that any group homomorphism $\phi : G \to G^l$ is either trivial or one to one

19. Let **G** be a group, show that $Z(G)$ the set of all elements in **G** which commutes with every element of **G,** is a normal subgroup of **G.**

20. Let p be a prime . Show that in a ring Zp , $(a + b)^p = a^p + b^p$ for all $a, b \in Z_p$

21. Show that $\phi : C \to M_2(R)$ given by $\phi(a + bi) = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ for $a, b \in R$ gives an isomorphism of C with the subring $\phi[c]$ of M_2 (R)

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Define isomorphism between two binary structures. Check whether $\langle \mathbb{Z}, + \rangle$ is isomorphic to $\langle 2\mathbb{Z}, + \rangle$ where $+$ is the usual addition.

23. 1. Let H be a subgroup of a group G. Let the relation $\sim_L$ be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H.$ Then show that $\sim_L$ is an equivalence relation on $G.$ What is the cell in the corresponding partition of G containing $a \in G?$
2. Let H be a subgroup of a group $G.$ Then define the left and right cosets of H containing $a \in G.$
3. Exhibit the left cosets and the right cosets of the subgroup $3\mathbb{Z}$ of $\mathbb{Z}.$

24. State and prove fundamental homomorphism theorem.

25. a) Prove that every field F is an integral domain.
b) Prove that every finite integral domain is a field.
c) Prove that Zp is a field if p is a prime.

(2×15=30)