22103521

QP CODE: 22103521

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, NOVEMBER 2022

Fifth Semester

CORE COURSE - MM5CRT03 - ABSTRACT ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

182F68AF

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. When can we say that a binary operation defined by a table is commutative?
2. Define general linear group of degree n.
3. Find the gcd of 32 and 24.
4. Define the octic group. Show that it is nonabelian.
5. Define a cycle. Show that the product of two cycles need not again be a cycle.
6. Define even and odd permutations. Give examples.
7. Define the Cartesian product of sets $S_1, S_2, \cdots, S_n$.. Write the number of elements in the Cartesian product of the sets $\{0, 1\}$ and $\{0, 1, 2\}$.
8. Define a group homomorphism with example.
9. If $\phi : G \to G'$ is a group homomorphism then show that $\phi(e) = e'$ where e and e' are identity elements of G and G' respectively.
10. Compute the product in the given ring a) (12) (16) in Z_{24} b) (20) (-8) in Z_{26}
11. Mark each of the following true or false.
    a) Q is an ideal in R
    b) The ring $Z/4Z$ and Z_4 are isomorphic

12. Define a factor ring

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Check whether $\langle \mathbb{C}, \cdot \rangle$ and $\langle \mathbb{R}, \cdot \rangle$ under usual multiplication are isomorphic.

14. a) Give the group table for the binary operation *addition modulo 2* on the set $\mathbb{Z}_2$.
    b) Give the group table for a binary operation $*$ on the set $\{e, a, b\}$.

15. Prove that a subset H of a group G is a subgroup of G if and only if
    a) H is closed under the binary operation of G,
    b) the identity element e of G is in H,
    c) for all $a \in H$ it is true that $a^{-1} \in H$ also.

16. Prove that for $n \geq 2$, the number of even permutations in S_n is the same as the number of odd permutations. Define the alternating group A_n on n letters.

17. Let H be a subgroup of a group G. Let the relation $\sim_R$ be defined on G by $a \sim_R b$ if and only if $ab^{-1} \in H$. Then show that $\sim_R$ is an equivalence relation on G. What is the cell in the corresponding partition of G containing $a \in G$?

18. Let **G** be a group, show that $Z(G)$ the set of all elements in **G** which commutes with every element of **G,** is a normal subgroup of **G.**

19. Define maximal normal subgroup of a group.Prove that **M** is a maximal normal subgroup of a group **G** if and only if the factor group G/M is simple.

20. Prove that the cancellation laws hold in a ring R if and only if R has no divisors of 0.

21. Prove that every field F is an integral domain.

(6×5=30)

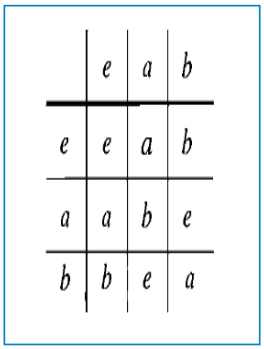
Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. Find all subgroups of $\mathbb{Z}_{18}$ and draw the subgroup diagram.

23. State and prove **Cayley's theorem**. Give the elements for the left regular representation and the group table of the group given by the group table



	e	a	b
e	e	a	b
a	a	b	e
b	b	e	a

24. State and prove fundamental homomorphism theorem.

25. a) Let p be a prime . Show that in a ring Z_p , $(a+b)^p = a^p + b^p$ for all $a, b \in Z_p$

b) Show that if a and b are nilpotent elements of a commutative ring , then a + b is also nilpotent.

c) Show that intersection of subrings of a ring R is again a subring of R

(2×15=30)