QP CODE: 22101060

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Reg No : ....

Name : ....

B.Sc DEGREE (CBCS) REGULAR / REAPPEARANCE EXAMINATIONS, APRIL 2022

Sixth Semester

CORE - MM6CRT04 - LINEAR ALGEBRA

Common for B.Sc Mathematics Model I & B.Sc Mathematics Model II Computer Science

2017 Admission Onwards

02BE78F7

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. *If the hyperbola $x^2 - y^2 = 1$ is rotated through 45^0 anticlockwise about the origin, what is the new equation.*
2. a)Define column rank of a matrix.

   b)Find the column rank of matrix. $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & -1 & 1 \\ 5 & 4 & -2 & 2 \end{bmatrix}$
3. *Define a left inverse and right inverse of a matrix*
4. *Prove that X= { (x,0) : x ε R} is a subspace of the vector space R^2*
5. Define a basis of a vector space V and write the canonical bases of R^n.
6. Define a bijective linear mapping. Give an example for a linear mapping that is injective but not surjective.
7. Define rank and nullity of a linear mapping. Also define Dimension Theorem for finite-dimensional vector spaces over a field F in terms of rank and nullity.
8. Determine the transition matrix from the ordered basis $\{(1,0,0,1),(0,0,0,1),(1,1,0,0),(0,1,1,0)\}$ of $\mathbb{R}^4$ to the natural ordered basis of $\mathbb{R}^4$.
9. Define a nilpotent linear mapping f on a vector space V of dimension n over a field F. What is meant by index of nilpotency of f.
10. Find the eigen values of A = $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & -1 \end{bmatrix}$
11. Define eigen value of a linear map and the eigen vector associated with it.
12. Define diagonalizable linear map and diagonalizable matrix.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. a)Prove that matrix multiplication is associative.

    b)Compute A^2 and A^3 where $A = \begin{bmatrix} 0 & a & a^2 \\ 0 & 0 & a \\ 0 & 0 & 0 \end{bmatrix}$

14. a)Prove that if the rows $x_1, x_2, \ldots\ldots, x_p$ are linearly independent, then none can be zero.

b) Prove that A = $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 1 & -1 & 1 \\ 5 & 4 & -2 & 2 \end{bmatrix}$ is linearly dependent.

15. *Prove that M_{mxn} (R) be the set of all mxn matrices is a vector space*

16. *Prove that $P(x) = 2 + x + x^2$, $q(x) = x + 2x^2$, $r(x) = 2 + 2x + 3x^2$ is linearly dependent.*

17. Define $Im\, f$ and $Ker\, f$ where f is a linear mapping from a vector space to a vector space. Write image and kernel for the differentiation map $D : \mathbb{R}_n[X] \to \mathbb{R}_n[X]$.

18. Suppose that the mapping $f : \mathbb{R}^3 \to \mathbb{R}^3$ is linear and such that $f(1,0,0) = (2,\ 3,\ -2), \quad f(1,1,0) = (4,\ 1,\ 4), \quad f(1,1,1) = (5,\ 1,\ -7).$ Find the matrix of f relative to the natural ordered basis of $\mathbb{R}^3$.

19. Define similar matrices. Show that if matrices A, B are similar then so are A', B'.

20. Determine the eigen values and their algebraic multiplicities of the linear mapping f: $R^3 \to R^3$ given by f(x,y,z) = (y + z, x + z, x + y)

21. For the nXn tridiagonal matrix An = $\begin{bmatrix} 2 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 1 & 2 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 2 & 1 & \ldots & 0 & 0 \\ . & . & . & . & \ldots.. & . & . \\ 0 & 0 & 0 & 0 & \ldots & 2 & 1 \\ 0 & 0 & 0 & 0 & \ldots & 1 & 2 \end{bmatrix}$ Prove that det An = n + 1.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22. a) Reduce the following matrix to Hermite form $\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 4 & 4 & 8 & 4 \\ 3 & 6 & 5 & 7 & 7 \end{bmatrix}$

b) Prove that by using elementary row operation, a non-zero matrix can be transformed to a row-echelon matrix.

c) Prove that every non-zero matrix A can be transformed to a Hermite matrix by using elementary row operations

23. a)Prove that the set W of complex matrices of the form $\begin{bmatrix} \alpha & \beta \\ -\bar{\beta} & -\alpha \end{bmatrix}$ is a real vector space of dimension 4.

b) If S is a subset of V then prove that S is a basis if and only if S is a maximal independent subset.

c) Let V be a finite dimensional vector space. If W is a subspace of V . Prove that W is of finite dimension and dim W ≤ dim V. If dim W = dim V iff W = V

24. Define linear mapping from a vector space to a vector space. Check whether $f : \mathbb{R}^3 \to \mathbb{R}^3$ given below are linear:

a) $f(x, y, z) = (x - 1, x, y)$.

b) $f(x, y, z) = (x + y, z, 0)$.

25. Show that $\{(1,1,1), (1,2,3), (1,1,2)\}$ is a basis of $\mathbb{R}^3$. If $f : \mathbb{R}^3 \to \mathbb{R}^3$ is linear and such that $f(1,1,1) = (1,1,1), \quad f(1,2,3) = (-1,-2,-3), \quad f(1,1,2) = (2,2,4)$, determine $f(x,y,z)$ for all $(x, y, z) \in \mathbb{R}^3$.

(2×15=30)